A SPORADIC SIMPLE GROUP OF B. FISCHER

OF ORDER 64,561,751,654,400

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ABSTRACT

In this thesis we study a sporadic simple group M(22) of order $64,561,751,654,400 = 2^{17}.3^{9}.5^{2}.7.11.13$ defined by B. Fischer [3].

In chapter I most of the material needed from other sources is presented. For example, in §1 we outline results from the paper "Finite groups generated by 3-transpositions" recently written by B. Fischer [3]. *

In chapter II we deal with the calculation of the character tables of PSU(5,2), PSU(6,2), $PS\Omega^{+}(6,3)$, $PS\Omega(7,3)$ and M(22).

In chapter III a characterization of M(22) by the structure of the centralizer of one of its involutions is given. The involution is not the involution central in the Sylow 2-subgroup but is a 3-transposition.

* (To appear in Inventiones Mathematicae.)

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§0 Introduction

The research presented in this thesis arises out of some recent work of B. Fischer on groups generated by 3-transpositions. All groups considered are finite.

The thesis naturally falls into 3 chapters.

The first chapter presents the results from other sources. Most of these are from Fischer's paper "Finite groups generated by 3-transpositions" but there are other sections dealing with Steiner systems, representations of the Mathieu groups over fields of characteristic 2, finite orthogonal groups and rank 3 permutation groups.

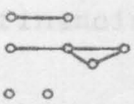
The second chapter outlines the methods used in calculating the character table of the smallest of the new simple groups of Fischer, M(22) or order $2^{17}.3^{9}.5^{2}.7.11.13$. The methods are not original although the use of the Eliott 4130 to reduce the considerable work involved in the calculation is new. As an aid to finding the character table of M(22) character tables of various classical groups PSU(5,2), PSU(6,2), $PS\Omega^{+}(6,3)$ and $PS\Omega(7,3)$ are determined.

The third chapter contains a characterization of M(22) by the structure of the centralizer of one of its involutions.

Most of the notation in the thesis is standard with the exception of the following :=

$d \in D$ d is a 3-transposition (§1).

D is a conjugacy class of 3-transpositions.

$t \in T$	t is the product of 2 commuting 3-transposition.
	T is the conjugacy class containing t.
$n \in N$	n is the product of 3 commuting 3-transpositions (which cannot be expressed as the product of 2 commuting 3-transpositions). N is the conjugacy class containing n.
D_d	$\{d_i \in D \mid d_i d = d d_i \text{ and } d_i \neq d\}$.
A_d	$\{d_i \in D \mid d_i d d_i = d d_i d \text{ and } d_i \neq d\}$.
	$D = \{d\} \cup D_d \cup A_d$.
L	A maximal set of commuting elements in D (§1).
	The group generated by involutions in D such that those points (= elements in D) joined do not commute and those not joined commute.
Σ_n	The symmetric group on n symbols.
$\chi_i^{(22)}$	An irreducible character of $M(22)$.
$\chi_i^{(5)}$	An irreducible character of $PSU(5,2)$.
$\chi_i^{(6)}$	An irreducible character of $PSU(6,2)$.
$\bar{\chi}_i^{(6)}$	An irreducible character of P^* (§1, §17).
$\chi_i^{(7)}$	An irreducible character of $PS\Omega(7,3)$.
$O(n,q)$, $\Omega(n,q)$ etc	See §4.

I would like to express my gratitude to Professor B. Fischer who was able to aid me greatly in my work on the character tables with his detailed knowledge of the groups.

I thank Mr. G.B. Elkington for his help in the work on conjugacy classes in classical groups.

Also I take this opportunity to thank Professor J.A. Green without whose help , advice and encouragement throughout my 3 years as a research student at the University of Warwick this work would never have been undertaken.

Finally my thanks go to the Association of Commonwealth Universities whose Scholarship supported me financially.

CHAPTER 1 PRELIMINARIES

§1 Groups generated by 3-transpositions

Definition 1.1 A conjugacy class D of a group G is called a class of conjugate 3-transpositions of G if :=

(i) $G = \langle d \mid d \in D \rangle$,

(ii) $o(d_1 d_2) = 1, 2$ or 3 for all $d_1, d_2 \in D$.

In [3] Fischer proves as main theorem :=

Theorem 1.2 Let D be a class of conjugate 3-transpositions of a finite group G satisfying :=

(i) $O_3(G) \leq Z(G)$; $O_2(G) \leq Z(G)$;

(ii) $G' = G''$.

Then $G/Z(G)$ is either :=

(I) Σ_n, the symmetric group on n symbols.

(II) $Sp(2n,2)$, the symplectic group over $GF(2)$.

(III) $P\Omega(n,q)$, $\overline{P\Omega}(n,q)$ n odd;
$P\Omega^{\pm}(n,q)$, $\overline{P\Omega}^{\pm}(n,q)$ n even; with q=2 or q=3. (See §4 for notation.)

(IV) $PSU(n,2)$, the projective special unitary group over $GF(4)$.

(V) M(22), M(23) or M(24), three new groups of orders $2^{17}.3^9.5^2.7.11.13$, $2^{18}.3^{13}.5^2.7.11.13.17.23$ and $2^{22}.3^{16}.5^2.7^3.11.13.17.23.29$, the first two being simple, the third containing a simple subgroup of index 2 and D is unique in all three groups.

Let G be one of the groups listed in Theorem 1.2.

Let S be a Sylow 2-subgroup of G.

We define L to be a maximal set of pairwise commuting elements in D, i.e. $L = D \cap S$. Then we define $|L|$ to be the D-width of G.

Lemma 1.3 If $G = PSU(6,2)$ then :=

(i) $|L| = 21$ [3:9.1,16.1].

(ii) $N_G(L)/C_G(L) = N_G(L)/\langle L\rangle \cong PSL(3,4) \cong M_{21}$. [3:16.1.17]. $N_G(L) = N_{21}$ (see §3).

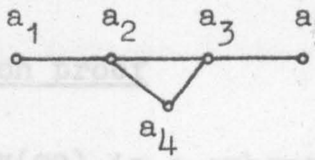
(iii) G contains 3 conjugacy classes of subgroups isomorphic to $P\Omega^-(6,3)$ [3:16.1.12].

Comments on proof

(i) The 3-transpositions in the unitary groups are transvections t(p) with respect to isotropic vectors p. i.e. $x \to xt(p) = x + (x,p)p$ for $x \varepsilon V$, where V is an n dimensional vector space over GF(4) and (,) is the non-singular hermitian form on V. 2 transvections commute if and only if $(p,q) = 0$. Therefore a set L corresponds to a maximal totally isotropic subspace, in this case a plane of order 21.

(ii) The action of M_{21} on $\langle L\rangle$ is as described in §3. $PSU(6,2) \geqslant N_{21}$ (see §3), i.e. there is a complement to $\langle L\rangle$ in $N_G(L)$. This is proved by showing that 512 of the 819 conjugates of $N_G(L)$ intersect $N_G(L)$ in a subgroup which intersects $\langle L\rangle$ trivially.

(iii) In $PSU(6,2)$ we take a fixed $d \varepsilon D$, $x \varepsilon A_d$. Then $K = \langle D_d \cap D_x \rangle \cong PSU(4,2)$. K is generated by 5 involutions $a_1, \ldots, a_5$ satisfying the relations :=



and $a_2^{a_4 a_3} \neq a_2^{a_4}$.

Now suppose $[d,a_5] = \{a_5, d, r_1, r_2, r_3\}$ where $[x,y]$ means the set of 5 elements in D corresponding to the points on the line through x and y in the geometry. Then we define :=

$$Q_i = \langle r_i, a_1, a_2, a_3, a_4 \rangle \qquad i=1,2,3.$$
$$R_i = \langle x, Q_i \rangle \qquad i=1,2,3.$$

Then the R_i, $i=1,2,3$ are representatives of the 3 conjugacy classes of subgroups isomorphic to $P\Omega^-(6,3)$.

<u>Lemma 1.4</u> If $G = M(22)$ then :=

(i) $C_G(d)/\langle d \rangle \cong PSU(6,2)$. We denote $C_G(d)$ by P^*. [3:17.1.2].

(ii) $|L| = 22$ [3:17.2.3(c)].

(iii) $|\langle L \rangle| = 2^{10}$ [3:17.2.3(d)].

(iv) $N_G(L)/C_G(L) = N_G(L)/\langle L \rangle \cong M_{22}$ [3:17.2.3(c)].

(v) G operates as a rank 3 group on the conjugacy class D [3:3.3.5].

(vi) G contains 2 conjugacy classes of subgroups isomorphic to $PS\Omega(7,3)$ [3:17.2.4].

(vii) G operates as a rank 3 group on the conjugates of a fixed $PS\Omega(7,3)$ [3:17.3.11].

(viii) G contains subgroups isomorphic to Σ_{10}.

(ix) If the involutions in an Σ_3 are in D then $C_G(\Sigma_3) \cong P\Omega^-(6,3)$.

Comments on proof

(i) M(22) is a subgroup of the automorphism group of a central extension of the graph $\Gamma(E)$ where E is the set of unitary transvections in PSU(6,2). Automatically the centralizer of a 3-transposition contains P* and order considerations show it is P*.

(ii) As $C_G(d)/\langle\pi\rangle \cong PSU(6,2)$ and the D-width of PSU(6,2) is 21 the D-width of M(22) is 22.

(iii) See §3.

(iv) Lemma: If $\langle D_d\rangle \cap N_G(L) = N_1$ acts n-ply transitively on $L \cap D_d$ then $N_G(L)$ acts n+1-ply transitively on L [3:3.3.9]. Therefore $N_G(L)/\langle L\rangle$ is a transitive extension of M_{21} and thus by a result of Luneberg [8] is isomorphic to M_{22}.

(v) See Lemma 1.5(ii) below.

(vi) M(22) is defined as the automorphism group of a width extension of $PS\Omega(7,3)$ with respect to the conjugacy class D and a subgroup $W \cong Sp(6,2)$ in $PS\Omega(7,3)$.

(vii) This is proved by finding all intersections of conjugates of a fixed $PS\Omega(7,3)$.

(ix) Proof uses Lemma 1.5(iii) and Lemma 1.3(iii).

NOTES (i) and (v) together imply that G has a rank 3 representation of degree 3510 on the conjugates of P*.

$N_G(L) = N_{22}$ (see §3). The proof as in Lemma 1.3(ii) consists of showing that there are 1024 conjugates of $N_G(L)$ meeting $N_G(L)$ in a subgroup which complements $\langle L\rangle$. ##

Lemma 1.5 Suppose G is a group listed in Theorem 1.2 then :=

(i) D_d is a conjugacy class of 3-transpositions of $\langle D_d\rangle$.

(ii) $\langle D_d\rangle$ is transitive on A_d.

(iii) $\langle D_d\rangle'/Z(\langle D_d\rangle')$ is simple implies that G acts transitively on the set of triples $\{x,d;f\}$ where $x,d,f \in D$ and $(\underset{x}{\circ}\overset{=}{\text{—}}\underset{d}{\circ}\ \ \underset{f}{\circ})$.

Definition $T = T(D) = \{d_1d_2 = d_2d_1 \neq 1 \mid d_1, d_2 \in D\}$.

Lemma 1.6 If G is one of the groups listed in Theorem 1.2 then :=

(i) $D \cap T = \varphi$ [3:4.1.1].

(ii) T is a class of conjugate elements of G [3:4.1.2].

(iii) If $d_1d_2 = d_3d_4 \in T$ for $d_i \in D$, $i=1,2,3,4$ then $\{d_1,d_2\} = \{d_3,d_4\}$ [3:4.1.2(b)].

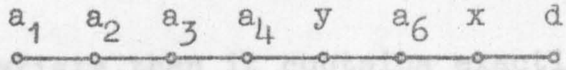
Lemma 1.7 If $G = PS\Omega(7,3)$ then :=

(i) G contains exactly 2 conjugacy classes of subgroups isomorphic to Σ_9 [3:15.3.8, 15.1.3].

(ii) G contains exactly 2 conjugacy classes of subgroups isomorphic to Sp(6,2) [3:15.3.11,15.1.3].

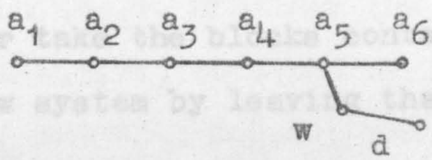
<u>Comments on proof</u>

(i) Fix $d \in D$ and $x \in A_d$. Then $C = \langle D_d \cap D_x \rangle \cong U_o S_1$ where $U_o < C$ is elementary abelian of order 3^4 and $S_1 \cong \Sigma_6$ $= \langle a_1, \ldots, a_5 \rangle$. $C_{D_d}(\langle a_1, \ldots, a_5 \rangle) = \{a_6\}$ and it is possible to replace a_5 by y so that we have elements satisfying the relations



and these are generators and relations of Σ_9.

(ii) We take $a_1, \ldots, a_6 \in D_d$ as in (i) and choose w so that



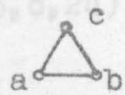
Then $\langle a_1, \ldots, a_6, w \rangle \cong W(E_7) \cong Sp(6,2)$. чч

<u>Lemma 1.8</u> Let D be a set of 3 transpositions of $\langle D \rangle$ and X a subgroup generated by 3 elements in D. Then X is an epimorphic image of one of the following groups :=

(i) $\Sigma_2 \times \Sigma_2 \times \Sigma_2$;

(ii) $\Sigma_2 \times \Sigma_3$;

(iii) Σ_4;

(iv) H where $H = \langle a,b,c \rangle$ 

$(b^c)^a \neq b^c$,

$|H| = 54$, $|Z(H)| = 3$, $|H'| = 3$ and H' is extraspecial of exponent 3. чч

§2 Steiner systems

Definition 2.1 A Steiner system $S(l,m,n)$ is an arrangement of n objects into blocks of size m so that each set of size l lies in exactly one block. Steiner systems do not exist for arbitrary l, m and n. (See Witt [12].)

If $S(l,m,n)$ exists then it contains exactly

$$\binom{n}{l}/\binom{m}{l} = \frac{n(n-1)\ldots.(n-l+1)}{m(m-1)\ldots.(m-l+1)} \text{ blocks.}$$

If $l > 2$ and $S(l,m,n)$ exists then $S(l-1,m-1,n-1)$ also exists. For take the blocks containing a given point and form the new system by leaving that point out.

Lemma 2.2 $S(5,8,24)$ exists and is unique. (Witt [12] proves its existence and uniqueness. Todd [10] actually lists the blocks as sets of 8 from the set $\{\infty\ 0\ 1\ 2\ldots22\}$.) By unique we mean := If Ω and Ω' are 2 sets of 24 points each forming a $S(5,8,24)$ then there is a 1-1 map between Ω and Ω' preserving the blocks.

The number of blocks in $S(5,8,24)$ is $\frac{24.23.22.21.20}{8.\ 7.\ 6.\ 5.\ 4}$ $= 759$.

The number of blocks containing a given 1 point is 253.

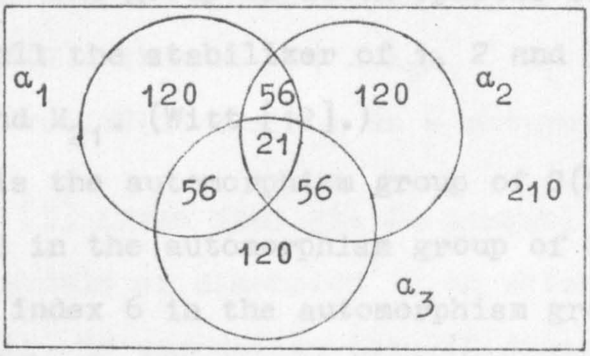
The number of blocks containing a given 2 points is 77.

The number of blocks containing a given 3 points is 21.

The number of blocks containing a given 4 points is 5.

The number of blocks containing a given 5 points is 1.

If $\alpha_1,\alpha_2,\alpha_3$ are arbitrary points from the 24 points of a S(5,8,24) then the following Venn diagram shows the number of blocks containing 1,2 and 3 of them.



<u>Definition 2.3</u> Let S(5,8,24) be a Steiner system on $P = \{\alpha_1,\alpha_2, \ldots ,\alpha_{24}\}$. Consider the fixed S(4,7,23), S(3,6,22) and S(2,5,21) obtained by leaving out the points α_{24}; α_{24} and α_{23}; α_{24},α_{23} and α_{22} respectively.

Then we define an 8-oval in S(4,7,23) to be a block in S(5,8,24) not containing α_{24}. There are 506 8-ovals in S(4,7,23).

A 7-oval in S(3,6,22) is a block in S(4,7,23) not containing α_{23}. There are 2.176 7-ovals in S(3,6,22).

A 6-oval in S(2,5,21) is a block in S(3,6,22) not containing α_{22}. There are 3.56 6-ovals in S(2,5,21).

<u>Lemma 2.4</u> The 8-ovals in S(4,7,23) are precisely the sets of 8 points, no 5 collinear in S(4,7,23). (Collinear means lies in a block.)

The 7-ovals in S(3,6,22) are the sets of 7 points no 4 collinear in S(3,6,22).

The 6-ovals in S(2,5,21) are the sets of 6 points no 3 collinear in S(2,5,21).

Proof Clearly all ovals have the property and a counting argument shows all sets with the given property are ovals.

Lemma 2.5 The group of automorphisms of $S(5,8,24)$ is M_{24}; we call the stabilizer of 1, 2 and 3 points in M_{24} M_{23}, M_{22} and M_{21}. (Witt [12].)

M_{23} is the automorphism group of $S(4,7,23)$. M_{22} is of index 2 in the automorphism group of $S(3,6,22)$ and M_{21} is of index 6 in the automorphism group of $S(2,5,21)$. (Witt [12].) $S(2,5,21)$ is actually a projective plane over $GF(4)$ and $M_{21} \cong PSL(3,4)$.

M_{23} is transitive on the set of 8-ovals.

M_{22} permutes the set of 7-ovals in 2 orbits of length 176 each.

M_{21} permutes the set of 6-ovals in 3 orbits of length 56 each. (Luneberg [8].)

§3 Modules for the Mathieu groups

M_{24}, M_{23} and M_{22} act naturally on 24, 23 and 22 symbols as a permutation group. Let $\Omega = \{\alpha_1, \alpha_2, \ldots, \alpha_{24}\}$. Let $F = GF(2)$. Then $F\Omega = \{ \Sigma \lambda_i \alpha_i \mid \lambda_i \in F, \alpha_i \in \Omega \}$ is a module over GF(2) for M_{24} in a natural way.

Todd [9] shows that the 24 dimensional module has a factor module of dimension 12 on which M_{24} acts irreducibly. We may define this 12 dimensional module in the following way :=

M_{24} is the automorphism group of S(5,8,24). We form an additive abelian group L_{12} with 24 generators $d_1, \ldots, d_{24}$ with the relations:

$2d_i = 0$.

$\sum_{j \in J} d_j = 0$ if and only if J is a union of blocks in S(5,8,24).

We may reduce this latter set of relations to

$\sum_{j \in J} d_j = 0$ if and only if J is a block.

Every element is a sum of $\leqslant 4$ d_i and as there are exactly 5 blocks through a given quadruple of points (Lemma 2.2), each element v which is the sum of 4 d's can be expressed as the sum of 4 d's in exactly 6 ways.

Hence L_{12} contains

element	number	
1	1	1
d_i	24	24
d_i+d_j	$\frac{24.23}{2.\ 1}$	276
$d_i+d_j+d_k$	$\frac{24.23.22}{3.2.\ 1}$	2024
$d_i+d_j+d_k+d_l$	$\frac{(24.23.22.21)}{(4.3.2.1.\ 6)}$	1771
		4096

Therefore $|L_{12}| = 2^{12}$.

M_{24} acts on L_{12} in a natural way because M_{24} permutes the i in such a way as to take each defining relation to another one. We denote by N_{24} the semi-direct product $L_{12}.M_{24}$. (L_{12} is defined in another way by Todd [9].)

We define L_{11} to have the same generators and relations as L_{12}, together with the relation $d_1 = 0$. i.e. L_{11} is a factor group of L_{12}. Then L_{11} is the abelian group generated by $d_2, \dots, d_{24}$ with the 759 relations

$$\sum_{j\varepsilon J} d_j = 0 \quad \text{if } J \text{ is an 8-oval in } S(4,7,23) \text{ or}$$
$$J \text{ is a block in } S(4,7,23).$$

Every element in L_{11} is the sum of 1, 2 or 3 d_j's.

Hence L_{11} contains :=

element	number	
1	1	1
d_i	23	23
d_i+d_j	$\frac{23.22}{2.\ 1}$	253
$d_i+d_j+d_k$	$\frac{23.22.21}{3.\ 2.\ 1}$	1771
		2048

Any sum of 4 d_j's = sum of 3 d_j's i.e. the complementary set in the unique block containing them. Therefore $|L_{11}| = 2^{11}$. M_{23} acts on L_{11}. We define $N_{23} = L_{11}.M_{23}$.

We define L_{10} by adding the relation $d_2 = 0$ to the relations in L_{11}. Then L_{10} is the abelian group generated by $d_3,\ldots,d_{24}$ with the 759+22 relations $2d_i = 0$ and

$\sum_{j\varepsilon J} d_j = 0$ if J is an 8-block in S(5,8,24) not containing d_1 and d_2 or J is a 7-oval in S(3,6,22) or J is a block in S(3,6,22).

The elements in L_{10} are sums of 1, 2 or 3 d_j's. If $n = d_1+d_2+d_3$ then n also $= d_4+d_5+d_6$ where $\{1,2,3,4,5,6\}$ is a block in S(3,6,22). These are the only expressions of n as a product of 3 d_j's.

Hence L_{10} contains:=

element	number	
1	1	1
d_i	22	22
d_i+d_j	$\frac{22.21}{2.\ 1}$	231
$d_i+d_j+d_k$	$(\frac{22.21.20}{3.\ 2.\ 1})\frac{1}{2}$	$\underline{770}$ $\underline{1024}$

Therefore $|L_{10}| = 2^{10}$. M_{22} acts on L_{10} and let $N_{22} = L_{10}.M_{22}$.

Finally L_9 contains :=

element	number	
1	1	1
d_i	21	21
d_i+d_j	$\frac{21.20}{2.\ 1}$	210
$d_i+d_j+d_k$	$(\frac{21.20.16}{3.\ 2.\ 1})\frac{1}{4}$	$\underline{280}$ $\underline{\underline{512}}$

$M_{21} = PSL(3,4)$ acts on L_9 and we let $N_{21} = L_9.M_{21}$.

§4 Orthogonal groups over finite fields

A general reference for this section is Artin [1].

Let F be a finite field GF(q) where $q = p^a$.

Let V be a vector space of dimension n over F.

Let $x \to x^J$ be an automorphism of order 1 or 2 of the field F.

Let (,) be a scalar product on V. i.e. (,) : $V \times V \to F$. (,) is said to be a sesquilinear form on V if:

$$\begin{aligned}(x_1+x_2,y) &= (x_1,y) + (x_2,y)\\(x,y_1+y_2) &= (x,y_1) + (x,y_2)\\(\lambda x,y) &= \lambda(x,y)\\(x,\lambda y) &= \lambda^J(x,y)\end{aligned}$$

for all x,y,x_1,x_2,y_1 and y_2 in V and λ in V.

If (,) is non-singular and $T : V \to V$ satisfies $(Tx,Ty) = (x,y)$ for all x,y in V then we say T is an ISOMETRY of (,).

If (,) is a non-singular sesquilinear form on V satisfying $(x,y) = (y,x)$ for all x,y in V then V is called an orthogonal space and the group of isometries of (,) is called an ORTHOGONAL group. We note that in this case J is the identity automorphism.

We say 2 sesquilinear forms are equivalent if they can be represented by the same matrix :=

$(e_i,e_j)_1 = (f_i,f_j)_2$ for different bases $\{e_i\},\{f_i\}$ of V.

Theorem 4.1 (a) Let V be a non-singular orthogonal space over F of dimension 2n. Then there are 2 non-equivalent forms on V with representatives :=

(1) $\underset{n}{\oplus} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(2) $\underset{n-1}{\oplus} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 0 & -\delta \end{bmatrix}$ where δ is a nonsquare in F.

(b) Let V be a non-singular orthogonal space over F of dimension 2n+1. Then there are 2 non-equivalent forms on V with representatives :=

(1) $\underset{n}{\oplus} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \end{bmatrix}$.

(2) $\underset{n}{\oplus} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} \delta \end{bmatrix}$ where δ is a nonsquare in F.

Definition 4.2 In case (a1) we denote the form by $\underline{\underline{0}}$ and the group of isometries as $O^+(2n,q)$.

In case (a2) we denote the form by $\underline{\underline{\omega}}$ and the group of isometries as $O^-(2n,q)$.

In case (b1) we denote the form by $\underline{\underline{1}}$ and the group of isometries as $O(2n+1,q)$.

In case (b2) we denote the form by $\underline{\underline{\delta}}$ and the group of isometries is the same as in case (b1) as $(\ ,\ )_{b2}$ is equivalent to $\delta(\ ,\ )_{b1}$.

$$|O^+(2n,q)| = 2.q^{n(n-1)}(q^n-1) \prod_{i=1}^{n-1} (q^{2i}-1).$$

$$|O^-(2n,q)| = 2.q^{n(n-1)}(q^n+1) \prod_{i=1}^{n-1} (q^{2i}-1).$$

$$|O(2n+1,q)| = 2.q^{n^2} \prod_{i=1}^{n} (q^{2i}-1).$$

Let V be a vector space of dimension n over F and (,) a non-singular sesquilinear form on V.

Let T be an isometry of (,) on V.

Let U = I - T. i.e. Ux = x - Tx for all x in V.

Define W_T = Image $U \subset V$.

(Tx,Ty) = (x,y). ∴ (Ux,Uy) = (Ux,y) + (x,Uy) for all x,y in V.

We define a form g_T on W_T as follows :=

$$g_T : W_T \times W_T \to F$$
$$: u,v \to [u,v] \quad \text{where}$$
$$[Ux,Uy] = (x,Uy).$$

This is a well defined map as if $Ux_1 = Ux_2$ then $(x_1,Uy) = (Ux_1,Uy) - (Ux_1,y) = (Ux_2,Uy) - (Ux_2,y) = (x_2,Uy)$

It is easy to check that g_T is a non-singular sesquilinear form on W_T.

Suppose V is an orthogonal space. Then [u,v] + [v,u] = (u,v) for all u,v in W_T.

Suppose that g_T has form A with respect to some basis then it has form S'AS with respect to another basis. $\det(S'AS) = \det A.(\det S)^2$ and hence T determines an element of $F^*/(F^*)^2$. We call this element the spinorial norm or discriminant of T.

Hence there are 2 homomorphisms of O(n,q). The first is the determinant map to $\{+1,-1\}$ and the second is the spinorial norm to $\{1,\delta\}$.

<u>Definition 4.3</u> $SO(n,q)$ is the subgroup of $O(n,q)$ of matrices of determinant 1.

$\Omega(n,q)$ is the subgroup of $O(n,q)$ of matrices with discriminant a square.

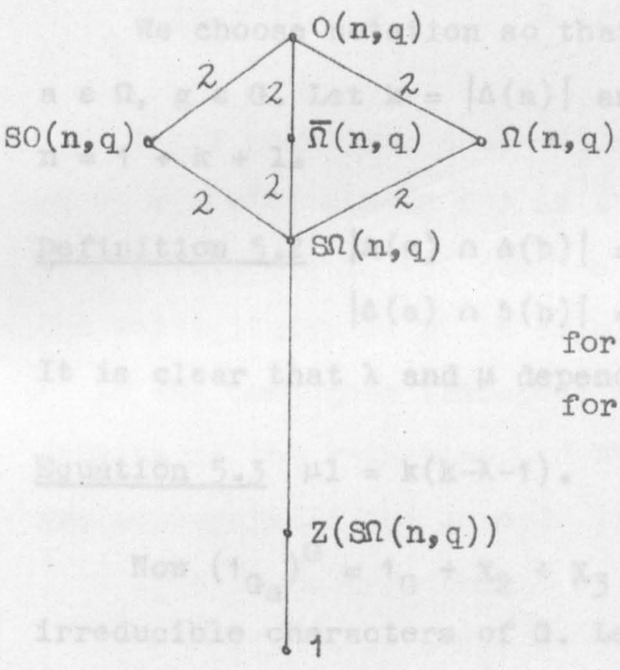
$S\Omega(n,q)$ is the intersection of these 2 subgroups.

$\overline{\Omega}(n,q)$ is the diagonal group.

$PS\overline{\Omega}(n,q) \cong S\overline{\Omega}(n,q)/Z(S\overline{\Omega}(n,q))$.

Note: We use $S\Omega^+(n,q)$ etc. for n even.

<u>Theorem 4.4</u> The inclusions are proper :=



$|Z(S\Omega(n,q))| = 1$

unless

for $O^+(2n,q)$ if $q^n-1 \equiv 0 \pmod 4$

for $O^-(2n,q)$ if $q^n+1 \equiv 0 \pmod 4$

when

$|Z(S\Omega(n,q))| = 2$.

§5 Rank 3 Permutation Groups

This section is taken from the paper by D.G. Higman [7].

Given a transitive group G of permutations of a finite set Ω, the number of G_a-orbits is independent of the particular $a \in \Omega$.

Let $n = |\Omega|$.

Definition 5.1 If G_a has exactly 3 orbits a, $\Delta(a)$ and $\Gamma(a)$ then we say that G is a rank 3 permutation group (strictly that G is a rank 3 permutation group on Ω).

We choose notation so that $\Delta(a)^g = \Delta(a^g)$ for all $a \in \Omega$, $g \in G$. Let $k = |\Delta(a)|$ and $l = |\Gamma(a)|$ so that $n = 1 + k + l$.

Definition 5.2 $|\Delta(a) \cap \Delta(b)| = \lambda$ for $b \in \Delta(a)$.
$|\Delta(a) \cap \Delta(b)| = \mu$ for $b \in \Gamma(a)$.

It is clear that λ and μ depend neither on a nor b.

Equation 5.3 $\mu l = k(k-\lambda-1)$.

Now $(1_{G_a})^G = 1_G + \chi_2 + \chi_3$ where χ_2 and χ_3 are irreducible characters of G. Let f_2 and f_3 be the degrees of χ_2 and χ_3. Then:

Equation 5.4 $$\{f_2, f_3\} = \frac{2k + (\lambda-\mu)(k+l) \mp \sqrt{d}(k+l)}{\mp 2\sqrt{d}}$$

where $d = (\lambda-\mu)^2 + 4(k-\mu)$.

CHAPTER 2 CHARACTER TABLES

§6 Conjugacy classes and character tables of PSU(5,2) and PSU(6,2)

Let V be a finite dimensional vector space over GF(q), where $q = r^2$. Let the dimension of the v.s. be n. A sesquilinear form on V [see §4] is unitary if $(x,y) = (y,x)^J$ for all $x,y \in V$. We call the group of isometries of such a form a unitary group. As all non-degenerate Hermitian forms on V are equivalent (Artin [1]) the unitary group $U(n,r^2)$ is essentially unique.

$$|U(n,r^2)| = r^{\frac{1}{2}(n-1)n} \prod_{i=1}^{n} (r^i-(-1)^i).$$

Let X stand for a non-singular linear transformation on V. $\varphi = \varphi(t)$ stands for an irreducible monic polynomial over GF(q), $\varphi \neq t$. $|\varphi|$ denotes the degree of φ and $m(\varphi^\mu)$ the multiplicity of φ^μ as an elementary divisor of X. X^* is the conjugate transpose of X, where conjugation is raising to the r-th power. 2 matrices are similar if they are conjugate in the general linear group.

If φ is an irreducible polynomial

$$\varphi(t) = \alpha_0 + \alpha_1 t + \ldots + t^k \text{ then}$$

$$\begin{aligned}\tilde{\varphi}(t) &= (\alpha_0^{-1})^J\ t^k\ \varphi^J(t^{-1}) \\ &= (\alpha_0^{-1})^J + \ldots + (\alpha_0^{-1}\alpha_1)^J\ t^{k-1} + t^k\end{aligned}$$

Theorem 6.1 (G.E. Wall [11])

(i) X is similar to an element of $U(n,q)$ if, and only if, $X \sim X^{*-1}$, i.e. $m(\varphi^{\mu}) = m(\tilde{\varphi}^{\mu})$ for all φ,μ.

(ii) 2 elements of $U(n,q)$ are conjugate in $U(n,q)$ if, and only if, they are similar.

(iii) The number of conjugacy classes in $U(n,q)$ is the coefficient of t^n in $\prod_{\lambda=1}^{\infty} \left(\frac{1+t^{\lambda}}{1-q^{\frac{1}{2}}t^{\lambda}}\right)$.

(iv) Let $X \in U(n,q)$. Write

$$A(\varphi^{\mu}) = \begin{cases} |U(m_{\mu},Q)| & (\varphi = \tilde{\varphi}) \\ |GL(m_{\mu},Q)|^{\frac{1}{2}} & (\varphi \neq \tilde{\varphi}) \end{cases}$$

$$B(\varphi) = Q^{\Sigma_{\mu<\nu}\mu m_{\mu} m_{\nu} + \frac{1}{2}\Sigma_{\mu}(\mu-1)m_{\mu}^2} \cdot \prod_{\mu} A(\varphi^{\mu})$$

where $Q = q^{|\varphi|}$, $m_{\mu} = m(\varphi^{\mu})$. Then the order of the centralizer of X, $|C_{U(n,q)}(X)| = \prod_{\varphi} B(\varphi)$. ♮♮

The unitary groups involved in M(22) are PSU(n,2), n=1,...,6. We list the orders of the relevant groups here:

n	$\|U(n,2)\|$	$\|SU(n,2)\|$	$\|PSU(n,2)\|$
4	$2^6.3^5.5$	$2^6.3^4.5$	$2^6.3^4.5$
5	$2^{10}.3^6.5.11$	$2^{10}.3^5.5.11$	$2^{10}.3^5.5.11$
6	$2^{15}.3^8.5.7.11$	$2^{15}.3^7.5.7.11$	$2^{15}.3^6.5.7.11$

These are groups over GF(4) i.e. q=4 and r=2 in Theorem 6.1, but there is no possibility of confusion in this inconsistency. Let $GF(4) = \{0,1,x,1+x\}$. Addition is mod 2 and $x^2=1+x$, $x(1+x)=1$.

The number of conjugacy classes in $U(n,2)$ is the coefficient of t^n in

$$\prod_{\lambda=1}^{\infty}\left(\frac{1+t^{\lambda}}{1-2t^{\lambda}}\right) = 1 + 3t + 9t^2 + 24t^3 + 60t^4 + 141t^5 + 324t^6 + \dots$$

These numbers are invaluable as checks for calculations.

The character table of $PSU(4,2)$ is well known and has been carefully checked by John Mackay. Therefore we deal with the conjugacy classes of $U(5,2)$ and $U(6,2)$ and thence those of $PSU(5,2)$ and $PSU(6,2)$.

The irreducible self-conjugate polynomials of degree $\leqslant 6$ over $GF(4)$ are :=

degree 1	$t+1, t+x, t+x+1$.
degree 2	none
degree 3	t^3+x, t^3+x+1.
degree 4	none
degree 5	$t^5+xt^4+t^3+t^2+(x+1)t+1$,
	$t^5+(x+1)t^4+t^3+t^2+xt+1$,
	$t^5+t^4+xt^3+t^2+xt+x$,
	$t^5+t^4+(x+1)t^3+t^2+(x+1)t+x+1$,
	$t^5+xt^4+xt^3+t^2+t+x$,
	$t^5+(x+1)t^4+(x+1)t^3+t^2+t+x+1$.
degree 6	none

The REMAINING irreducible polynomials of degree 1,2 and 3 arranged in conjugate pairs are as follows:=

degree 1 none

degree 2 $\quad t^2+xt+1,\ t^2+(x+1)t+1;$

$t^2+t+x+1,\ t^2+(x+1)t+x+1;$

$t^2+t+x,\ t^2+xt+x.$

degree 3 $\quad t^3+t+1,\ t^3+t^2+1;$

$t^3+xt+1,\ t^3+(x+1)t^2+1;$

$t^3+(x+1)t+1,\ t^3+xt^2+1;$

$t^3+t^2+t+x+1,\ t^3+(x+1)t^2+(x+1)t+x+1;$

$t^3+t^2+t+x,\ t^3+xt^2+xt+x.$

$t^3+xt^2+t+x+1,\ t^3+(x+1)t^2+xt+x+1;$

$t^3+xt^2+(x+1)t+x,\ t^3+(x+1)t^2+t+x;$

$t^3+t^2+xt+x+1,\ t^3+xt^2+(x+1)t+x+1;$

$t^3+t^2+(x+1)t+x,\ t^3+(x+1)t^2+xt+x;$

§7 Conjugacy classes in PSU(5,2)

Using Theorem 6.1 it is now a routine matter to list the conjugacy classes of U(5,2) and determine which of these conjugacy classes lie in SU(5,2)=PSU(5,2). It then remains to determine which of these conjugacy classes split into 3 conjugacy classes in SU(5,2), i.e. those for which $|C_{U(5,2)}(X)| = |C_{SU(5,2)}(X)|$.

The conjugacy classes in U(5,2) which lie in SU(5,2) are listed in Table 1, none of them split as $|Z(U(5,2))| = 3$ and $|Z(SU(5,2))| = 1$.

TABLE 1

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{SU(5,2)}(X)\rvert$
t+1	5	$2^{10}.3^{6}.5.11$		$2^{10}.3^{5}.5.11$
$(t+1)^2$ t+1	1 3	3 $2^3.3^4$	$4^{3+\frac{1}{2}}$	$2^{10}.3^{4}$
$(t+1)^2$ t+1	2 1	2.3^2 3	$4^{2+\frac{1}{2}}.2^2$	$2^{9}.3^{2}$
$(t+1)^3$ t+1	1 2	3 2.3^2	$4^{2+\frac{1}{2}}.2$	$2^{7}.3^{2}$
$(t+1)^3$ $(t+1)^2$	1 1	3 3	$4^{2+\frac{1}{2}(1+2)}$	$2^{7}.3$
$(t+1)^4$ t+1	1 1	3 3	$4^{1+\frac{1}{2}}.3$	$2^{5}.3$
$(t+1)^5$	1	3	$4^{\frac{1}{2}}.4$	2^{4}

TABLE 1 (Contd.)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{SU(5,2)}(X)\rvert$
t+x t+x+1	4 1	$2^6.3^5.5$ 3		$2^6.3^5.5$
t+x+1 t+x	4 1	$2^6.3^5.5$ 3		$2^6.3^5.5$
$(t+x)^2$ t+x t+x+1	1 2 1	3 2.3^2 3	$4^{2+\frac{1}{2}}$	$2^6.3^3$
$(t+x+1)^2$ t+x+1 t+x	1 2 1	3 2.3^2 3	$4^{2+\frac{1}{2}}$	$2^6.3^3$
$(t+x)^2$ t+x+1	2 1	2.3^2 3	$4^{\frac{1}{2}}.2^2$	$2^5.3^2$
$(t+x+1)^2$ t+x	2 1	2.3^2 3	$4^{\frac{1}{2}}.2^2$	$2^5.3^2$
$(t+x)^3$ t+x t+x+1	1 1 1	3 3 3	$4^{1+\frac{1}{2}}.2^2$	$2^4.3^2$
$(t+x+1)^3$ t+x+1 t+x	1 1 1	3 3 3	$4^{1+\frac{1}{2}}.2$	$2^4.3^2$
$(t+x)^4$ t+x+1	1 1	3 3	$4^{\frac{1}{2}}.3$	$2^3.3$
$(t+x+1)^4$ t+x	1 1	3 3	$4^{\frac{1}{2}}.3$	$2^3.3$
t+x t+1	3 2	$2^3.3^4$ 2.3^2		$2^4.3^5$

TABLE 1 (Contd.)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{SU(5,2)}(X)\rvert$
t+x+1 t+1	3 2	$2^3.3^4$ 2.3^2		$2^4.3^5$
t+x $(t+1)^2$	3 1	$2^3.3^4$ 3	$4^{\frac{1}{2}}$	$2^4.3^4$
t+x+1 $(t+1)^2$	3 1	$2^3.3^4$ 3	$4^{\frac{1}{2}}$	$2^4.3^4$
$(t+x)^2$ t+x t+1	1 1 2	3 3 2.3^2	$4^{1+\frac{1}{2}}$	$2^4.3^3$
$(t+x+1)^2$ t+x+1 t+1	1 1 2	3 3 2.3^2	$4^{1+\frac{1}{2}}$	$2^4.3^3$
$(t+x)^2$ t+x $(t+1)^2$	1 1 1	3 3 3	$4^{1+\frac{1}{2}}$ $4^{\frac{1}{2}}$	$2^4.3^2$
$(t+x+1)^2$ t+x+1 $(t+1)^2$	1 1 1	3 3 3	$4^{1+\frac{1}{2}}$ $4^{\frac{1}{2}}$	$2^4.3^2$
$(t+x)^3$ t+1	1 2	3 2.3^2	$4^{\frac{1}{2}}.2$	$2^3.3^2$
$(t+x+1)^3$ t+1	1 2	3 2.3^2	$4^{\frac{1}{2}}.2$	$2^3.3^2$
$(t+x)^3$ $(t+1)^2$	1 1	3 3	$4^{\frac{1}{2}}.2$ $4^{\frac{1}{2}}$	$2^3.3$
$(t+x+1)^3$ $(t+1)^2$	1 1	3 3	$4^{\frac{1}{2}}.2$ $4^{\frac{1}{2}}$	$2^3.3$

TABLE 1 (Contd.)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{SU(5,2)}(X)\rvert$
$t+1$ $t+x$ $t+x+1$	3 1 1	$2^3.3^4$ 3 3		$2^3.3^5$
$(t+1)^2$ $t+1$ $t+x$ $t+x+1$	1 1 1 1	3 3 3 3	$4^{1+\frac{1}{2}}$	$2^3.3^3$
$(t+1)^3$ $t+x$ $t+x+1$	1 1 1	3 3 3	$4^{\frac{1}{2}}.2$	$2^2.3^2$
$t+x$ $t+x+1$ $t+1$	2 2 1	2.3^2 2.3^2 3		$2^2.3^4$
$t+x$ $(t+x+1)^2$ $t+1$	2 1 1	2.3^2 3 3	$4^{\frac{1}{2}}$	$2^2.3^3$
$t+x+1$ $(t+x)^2$ $t+1$	2 1 1	2.3^2 3 3	$4^{\frac{1}{2}}$	$2^2.3^3$
$(t+x)^2$ $(t+x+1)^2$ $t+1$	1 1 1	3 3 3	$4^{\frac{1}{2}}$ $4^{\frac{1}{2}}$	$2^2.3^2$
t^3+x $t+x$	1 2	3^2 2.3^2		2.3^3
t^3+x+1 $t+x+1$	1 2	3^2 2.3^2		2.3^3

TABLE 1 (Contd.)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{SU(5,2)}(X)\rvert$
t^3+x	1	3^2		
$(t+x)^2$	1	3	$4^{\frac{1}{2}}$	2.3^2
t^3+x+1	1	3^2		
$(t+x+1)^2$	1	3	$4^{\frac{1}{2}}$	2.3^2
t^3+x	1	3^2		
$t+1$	1	3		
$t+x+1$	1	3		3^3
t^3+x+1	1	3^2		
$t+1$	1	3		
$t+x$	1	3		3^3
t^2+xt+1 $t^2+(x+1)t+1$	1	3.5		
$t+1$	1	3		3.5
$t^2+t+x+1$ $t^2+(x+1)t+x+1$	1	3.5		
$t+x+1$	1	3		3.5
t^2+t+x t^2+xt+x	1	3.5		
$t+x$	1	3		3.5
$t^5+xt^4+t^3+t^2+(x+1)t+1$	1	3.11		11
$t^5+(x+1)t^4+t^3+t^2+xt+1$	1	3.11		11

§8 Characters of PSU(5,2)

We determine the characters of PSU(5,2), $\chi_i^{(5)}$ $i = 1,\ldots,47$, mainly by using the characters of the subgroup of index 176, namely $PSU(4,2) \times C_3$. First, however we calculate the characters of the three rank 3 representations of degrees 165,176 and 297, τ_{165}, τ_{176} and τ_{297}.

The projective geometry PG(5,4) contains $1+4+4^2+4^3+ +4^4 = (4^5-1)/(4-1) = 341$ points.

If $\underset{\sim}{x}$ is a point we define the length of $\underset{\sim}{x} = l(\underset{\sim}{x})$ to be $\underset{\sim}{x}^*\underset{\sim}{x} = \underset{\sim}{x}^{TJ}\underset{\sim}{x}$. e.g. if $\underset{\sim}{x} = \begin{bmatrix} 1 \\ x \\ 1+x \\ 0 \\ 0 \end{bmatrix}$ then $l(\underset{\sim}{x}) = 1.1+x.(1+x)+(1+x).x = 1$

Clearly $l(\underset{\sim}{x}) = 1$ if and only if $\underset{\sim}{x}$ has an odd number of non-zero coordinates and $l(\underset{\sim}{x}) = 0$ if and only if $\underset{\sim}{x}$ has an even number of non-zero coordinates.

Therefore the number of non-isotropic points, those $\underset{\sim}{x}$ with $l(\underset{\sim}{x}) = 1$ is $5 + \binom{5}{3}.3^2 + \binom{5}{5}.3^4 = 176$.

The number of isotropic points is $\binom{5}{2}.3 + \binom{5}{4}.3^3 = 165$.

If $\underset{\sim}{x}$ is an isotropic point and $\underset{\sim}{y}$ is an isotropic point such that $\underset{\sim}{x}^*\underset{\sim}{y} = 0$ then the line joining $\underset{\sim}{x}$ and $\underset{\sim}{y}$ is totally isotropic as $(\lambda\underset{\sim}{x} + \mu\underset{\sim}{y})^*(\lambda\underset{\sim}{x} + \mu\underset{\sim}{y}) = 0$ for all $\lambda,\mu \in GF(4)$.

Clearly PSU(5,2) is transitive on the isotropic points.

Consider $\underset{\sim}{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Then the number of $\underset{\sim}{y}$, isotropic, such that $\underset{\sim}{x}^*\underset{\sim}{y} = 0$ is

$3.3 + 1 + 3.9 = 37$ of type

$$\begin{bmatrix}0\\0\\a\\b\\c\end{bmatrix} \quad \begin{bmatrix}1\\1\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}1\\1\\a\\b\\c\end{bmatrix}$$

Therefore the number of totally isotropic lines is

$$\frac{165 \times (37-1)}{5 \times 4} = 297.$$

It is easy to see that the characters τ_{176}, τ_{165} and τ_{297} are all characters of rank 3 representations.

The character of the representation on 341 points, $\tau_{341} = \tau_{176} + \tau_{165}$ can be calculated using only the elementary divisors of the element, as the number of fixed points of an element acting on a PG(n,q) depends only on the conjugacy class of the element in GL(n,q).

e.g. if $X \in PSU(5,2) = SU(5,2)$, where X has elementary divisors $(t+x)^2, t+x, t+1, t+1$, then X is similar to the matrix $\begin{bmatrix} x & 1 & & & \\ & x & & & \\ & & x & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$. The fixed vectors are

$$\begin{bmatrix}1\\1\\0\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\x\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\1+x\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\1\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\x\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\1+x\end{bmatrix} \begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix}$$

Therefore $\tau_{341}(X) = 10$.

The stabilizer of a non-isotropic point is isomorphic to $PSU(4,2) \times C_3$. Consider point $\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$.

$$\text{Hence } \tau_{176} = (1_{PSU(4,2) \times C_3})^{PSU(5,2)}.$$

$$\tau_{165} = \tau_{341} - \tau_{176}.$$

We now apply the rank 3 results from §5.

τ_{165}: $n=165$ $k=36$ $l=128$

$\lambda=3$ (Other 3 points on a totally isotropic line)

$$\mu l = k(k-\lambda-1) \quad \text{(Equation 5.3)}$$

Therefore $\mu = 36.32/128 = 9.$

Therefore $d = (\lambda-\mu)^2 + 4(k-\mu) = 144.$

$$\{f_2, f_3\} = \frac{2k + (\lambda-\mu)(k+l) \mp \sqrt{d}(k+l)}{\mp 2\sqrt{d}} \quad \text{(Equation 5.4)}$$

$$= \frac{72 - 6.164 \mp 12.164}{\mp 2.12} = \{120, 44\}.$$

τ_{176}: $n=176$ $k=40$ $l=135$ $\lambda=12$

Therefore $\mu = 40.27/135 = 8.$

$$d = 16 + 4.32 = 144.$$

$$\{f_2, f_3\} = \frac{80 + 4.175 \mp 12.175}{\mp 2.12} = \{120, 55\}.$$

The character τ_{176} is non-zero on all except 8 conjugacy classes of PSU(5,2). Hence by restricting to the subgroup PSU(4,2) x C_3 it is possible to calculate the value of the 3 irreducible characters of degrees 44, 55 and 120 on all the conjugacy classes except the 8 where τ_{176} vanishes. (The character of degree 120 is the same character in both τ_{165} and τ_{176}.)

2 of the 8 conjugacy classes are elements of order 11 and the values on these classes are clear by congruences mod 11. There are very few possibilities for the values

of the 3 characters on the remaining 6 conjugacy classes and orthogonality relations determine these.

τ_{297} splits as $\chi_1^{(5)}$ and 2 characters of degrees 120 and 176. The character of degree 120 is the one already known.

Finally using the Elliott 4100 computer at Warwick University the 60 induced characters, induced from PSU(4,2) x C_3, were calculated together with the matrix of their scalar products. The remaining characters were then easily determined by solving the sets of linear equations resulting from the matrix of scalar products.

The character table of PSU(5,2) is presented as TABLE 2.

TABLE 2

CHARACTER TABLE OF PSU(5, 2)

ORDER = 13,685,760 = $2^{10}.3^5.5.11$

Order(x)	1	2	2	4	4	4	8	3	3	6	6	6	6	12	12	12	12	3	3	6	6	6	6	6	6	12	12	12	12	3	6	12	6	6	6	3	9	9	18	18	9	9	5	15	15	11	11
$\lvert C_{PSU(5,2)}(x)\rvert$	$2^{10}.3^5.5.11$	$2^{10}.3^4$	$2^9.3^2$	$2^7.3^2$	$2^7.3$	$2^5.3$	2^4	$2^6.3^5.5$	$2^6.3^5.5$	$2^6.3^3$	$2^6.3^3$	$2^5.3^2$	$2^5.3^2$	$2^4.3^2$	$2^4.3^2$	$2^3.3$	$2^3.3$	$2^4.3^5$	$2^4.3^5$	$2^4.3^4$	$2^4.3^4$	$2^4.3^3$	$2^4.3^3$	$2^4.3^2$	$2^4.3^2$	$2^3.3^2$	$2^3.3^2$	$2^3.3$	$2^3.3$	$2^3.3^5$	$2^3.3^3$	$2^2.3^2$	$2^2.3^2$	$2^2.3^3$	$2^2.3^3$	$2^2.3^4$	2.3^3	2.3^3	2.3^2	2.3^2	3^3	3^3	3.5	3.5	3.5	11	11
$\lvert Cl(x)\rvert$	1	165	2970	11880	35640	142560	855360	176	176	7920	7920	47520	47520	95040	95040	570240	570240	3520	3520	10560	10560	31680	31680	95040	95040	190080	190080	570240	570240	7040	63360	380160	380160	126720	126720	42240	253440	253440	760320	760320	506880	506880	912384	912384	912384	1244160	1244160
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	10	-6	2	2	-2	-2	0	-5	-5	3	3	-1	-1	-1	-1	1	1	1	1	3	3	-3	-3	-1	-1	-1	-1	1	1	4	0	2	2	0	0	-2	-2	-2	0	0	1	1	0	0	0	-1	-1
χ_3	11	-5	3	3	-1	-1	1	-1 -6ω	5 6ω	-1 2ω	-3 2ω	-1 -2ω	1 2ω	-1 -2ω	1 2ω	-1	-1	-1 3ω	-4 -3ω	1 3ω	-2 -3ω	-1 -ω	ω	1 -ω	2 ω	-1 ω	-2 -ω	1 ω	-ω	2	-2	0	0	2 2ω	-2ω	2	-ω	1 ω	ω	-1 -ω	-ω	1 ω	1	ω	-1 -ω	0	0
χ_4	11	-5	3	3	-1	-1	1	5 6ω	-1 -6ω	-3 -2ω	-1 2ω	1 2ω	-1 -2ω	1 2ω	-1 -2ω	-1	-1	-4 -3ω	-1 3ω	-2 -3ω	1 3ω	ω	-1 -ω	2 ω	1 -ω	-2 -ω	-1 ω	-ω	1 ω	2	-2	0	0	-2ω	2 2ω	2	1 ω	-ω	-1 -ω	1 ω	1 ω	-ω	1	-1 -ω	ω	0	0
χ_5	44	12	-4	4	4	0	0	14	14	6	6	2	2	-2	-2	0	0	-1	-1	3	3	3	3	-1	-1	1	1	1	1	5	-3	1	2	0	0	2	2	2	0	0	-1	-1	-1	-1	-1	0	0
χ_6	55	23	7	-1	-1	3	-1	10	10	2	2	-2	-2	2	2	0	0	1	1	5	5	5	5	1	1	-1	-1	-1	-1	10	2	2	1	-1	-1	1	1	1	-1	-1	1	1	0	0	0	0	0
χ_7	55	7	-1	7	3	-1	1	-10 -15ω	-5 15ω	-4 -7ω	3 7ω	-4 -3ω	-1 3ω	ω	-1 -ω	-ω	1 ω	-2 -6ω	4 6ω	-2	-2	2 2ω	-2ω	2	2	-2ω	2 2ω	0	0	1	1	1	-1	-1 2ω	-3 -2ω	1	-1 -ω	ω	-1 -ω	ω	-1 -ω	ω	0	0	0	0	0
χ_8	55	7	-1	7	3	-1	1	-5 15ω	-20 -15ω	3 7ω	-4 -7ω	-1 3ω	-4 -3ω	-1 -ω	ω	1 ω	-ω	4 6ω	-2 6ω	-2	-2	-2ω	2 2ω	2	2	2 2ω	-2ω	0	0	1	1	1	-1	-3 -2ω	-1 2ω	1	ω	-1 -ω	ω	-1 -ω	ω	-1 -ω	0	0	0	0	0
χ_9	66	18	10	2	-2	2	0	-15 -9ω	-6 9ω	1 -ω	2 ω	1 3ω	-2 -3ω	-3 -ω	-2 ω	1 ω	-ω	-6 -9ω	3 9ω	-6 -3ω	-3 3ω	-2 -ω	-1 ω	-2 -3ω	1 3ω	-ω	1 ω	ω	-1 -ω	3	3	-1	1	1 2ω	-1 -2ω	3	0	0	0	0	0	0	1	-1 -ω	ω	0	0
χ_{10}	66	18	10	2	-2	2	0	-6 9ω	-15 -9ω	2 ω	1 -ω	-2 -3ω	1 3ω	-2 ω	-3 -ω	-ω	1 ω	3 9ω	-6 -9ω	-3 3ω	-6 -3ω	-1 ω	-2 -ω	1 3ω	-2 -3ω	1 ω	-ω	-1 -ω	ω	3	3	-1	1	-1 -2ω	1 2ω	3	0	0	0	0	0	0	1	ω	-1 -ω	0	0
χ_{11}	110	-2	-10	6	-6	2	0	-25	-25	7	7	-1	-1	3	3	-1	-1	2	2	-2	-2	-2	-2	2	2	0	0	0	0	2	-2	0	2	-2	-2	2	-1	-1	1	1	-1	-1	0	0	0	0	0
χ_{12}	110	30	6	6	2	2	0	-25 -30ω	5 30ω	-9 -6ω	-3 6ω	-1 -2ω	1 2ω	-1 -2ω	1 2ω	-1	-1	-4 -3ω	-1 3ω	-6 -9ω	3 9ω	0 -3ω	3 3ω	-2 -ω	-1 ω	2 ω	1 -ω	0 -ω	1 ω	8	0	0	0	0	0	-4	-2 -2ω	0 2ω	0	0	1 ω	-ω	0	0	0	0	0
χ_{13}	110	30	6	6	2	2	0	5 30ω	-25 -30ω	-3 6ω	-9 -6ω	1 2ω	-1 -2ω	1 2ω	-1 -2ω	-1	-1	-1 3ω	-4 -3ω	3 9ω	-6 -9ω	3 3ω	-3ω	-1 ω	-2 -ω	1 -ω	2 ω	1 ω	-ω	8	0	0	0	0	0	-4	0 2ω	-2 -2ω	0	0	ω	1 ω	0	0	0	0	0
χ_{14}	110	-34	6	-2	2	-2	0	-10 -15ω	5 15ω	-2 ω	-3 -ω	2 ω	1 -ω	-2 -3ω	1 3ω	ω	-1 -ω	-1 -6ω	5 6ω	5 6ω	-1 -6ω	-5 -2ω	-3 2ω	1 2ω	-1 -2ω	1	1	-1	-1	11	-1	1	0	-2 -2ω	2ω	2	-ω	1 ω	-ω	1 ω	-ω	1 ω	0	0	0	0	0
χ_{15}	110	-34	6	-2	2	-2	0	5 15ω	-10 -15ω	-3 -ω	-2 ω	1 -ω	2 ω	1 3ω	-2 -3ω	-1 -ω	ω	5 6ω	-1 -6ω	-1 -6ω	5 6ω	-3 2ω	-5 -2ω	-1 -2ω	1 2ω	1	1	-1	-1	11	-1	1	0	2ω	-2 -2ω	2	1 ω	-ω	1 ω	-ω	1 ω	-ω	0	0	0	0	0
χ_{16}	120	24	8	8	0	0	0	30	30	6	6	2	2	2	2	0	0	12	12	6	6	0	0	2	2	2	2	0	0	3	3	-1	-1	3	3	3	0	0	0	0	0	0	0	0	0	-1	-1
χ_{17}	165	-27	5	5	5	-3	1	30	30	6	6	2	2	2	2	0	0	3	3	-9	-9	3	3	-1	-1	-1	-1	-1	-1	3	3	-1	2	0	0	-6	0	0	0	0	0	0	0	0	0	0	0
χ_{18}	176	-16	16	0	0	0	0	-4	-4	-4	-4	4	4	0	0	0	0	14	14	2	2	2	2	-2	-2	0	0	0	0	-4	-4	0	1	-1	-1	5	-1	-1	-1	-1	-1	-1	1	1	1	0	0
χ_{19}	220	-36	-4	4	-4	0	0	40 30ω	10 -30ω	-8 -10ω	2 10ω	2ω	-2 -2ω	-2ω	2 2ω	0	0	10 3ω	7 -3ω	-6 -3ω	-3 3ω	-2 -ω	-1 ω	-2 -ω	-3 -ω	ω	-1 -ω	-ω	1 ω	1	3	1	-1	1 2ω	-1 -2ω	1	2 2ω	-2ω	0	0	-1 -ω	ω	0	0	0	0	0
χ_{20}	220	-36	-4	4	-4	0	0	10 -30ω	40 30ω	2 10ω	-8 -10ω	-2 -2ω	2ω	2 2ω	-2ω	0	0	7 -3ω	10 3ω	-3 3ω	-6 -3ω	-1 ω	-2 -ω	-3 -ω	-2 -ω	-1 -ω	ω	1 ω	-ω	7	3	1	-1	-1 -2ω	1 2ω	1	-2ω	2 2ω	0	0	ω	-1 -ω	0	0	0	0	0
χ_{21}	220	-4	12	4	4	0	0	10 30ω	-20 -30ω	2 6ω	-4 -6ω	-2 2ω	-4 -2ω	2 2ω	0 -2ω	0	0	-17 -15ω	-2 15ω	-1 -3ω	2 3ω	-1 -3ω	2 3ω	-1 ω	-2 -ω	-1 -ω	ω	-1 -ω	ω	-5	-1	1	0	2	2	4	-1 -ω	ω	1 ω	-ω	-1 -ω	ω	0	0	0	0	0
χ_{22}	220	-4	12	4	4	0	0	-20 -30ω	10 30ω	-4 -6ω	2 6ω	-4 -2ω	-2 2ω	-2ω	2 2ω	0	0	-2 15ω	-17 -15ω	2 3ω	-1 -3ω	2 3ω	-1 -3ω	-2 -ω	-1 ω	ω	-1 -ω	ω	-1 -ω	-5	-1	1	0	2	2	4	ω	-1 -ω	-ω	1 ω	ω	-1 -ω	0	0	0	0	0
χ_{23}	264	-24	-8	8	0	0	0	-24 -54ω	30 54ω	-8 2ω	-10 -2ω	-2ω	2 2ω	2ω	-2 -2ω	0	0	-6	-6	6 6ω	-6ω	-2 4ω	2 4ω	2 2ω	-2ω	2ω	-2 -2ω	0	0	3	3	-1	1	1 2ω	-1 -2ω	3	0	0	0	0	0	0	-1	-ω	1 ω	0	0
χ_{24}	264	-24	-8	8	0	0	0	30 54ω	-24 -54ω	-10 -2ω	-8 2ω	2 2ω	-2ω	-2 -2ω	2ω	0	0	-6	-6	-6ω	6 6ω	2 4ω	-2 -4ω	-2ω	2 2ω	-2 -2ω	2ω	0	0	3	3	-1	1	-1 -2ω	1 2ω	3	0	0	0	0	0	0	-1	1 ω	-ω	0	0
χ_{25}	320	-64	0	0	0	0	0	-40	-40	8	8	0	0	0	0	0	0	-4	-4	8	8	-4	-4	0	0	0	0	0	0	14	2	0	0	2	2	-4	-1	-1	-1	-1	-1	-1	0	0	0	1	1
χ_{26}	330	-6	2	10	-2	2	0	60 45ω	15 -45ω	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-3	-3	1	-1	-1 -2ω	1 2ω	-3	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	330	-6	2	10	-2	2	0	15 -45ω	60 45ω	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-3	-3	1	-1	1 2ω	-1 -2ω	-3	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	440	88	8	-8	0	0	0	-10	10	-2	-2	2	2	-2	-2	0	0	8	8	-2	-2	4	4	2	2	-2	-2	0	0	17	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1	0	0	0	0	0
χ_{29}	440	-40	8	8	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-1	-1	-1	-1	-1	-1	-1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
χ_{30}	440	-40	8	8	0	0	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-1	-1	-1	-1	-1	-1	-1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0	0
χ_{31}	495	-81	15	-1	-1	-1	-1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	495	-81	15	-1	-1	-1	-1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	495	63	-9	-1	-5	-1	1	45 45ω	-45ω	-3 -3ω	3ω	-3 -3ω	3ω	1 ω	-ω	1 ω	-ω	-1 -9ω	9ω	9 9ω	-9ω	3 3ω	-3ω	-3 -3ω	3ω	1 ω	-ω	-1 -ω	ω	9	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	495	63	-9	-1	-5	-1	1	-45ω	45 45ω	3ω	-3 -3ω	3ω	-3 -3ω	-ω	1 ω	-ω	1 ω	9ω	-9 -9ω	-9ω	9 9ω	-3ω	3 3ω	3ω	-3 -3ω	-ω	1 ω	-1 ω	-1 -ω	9	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	660	84	20	-4	4	0	0	30	30	6	6	2	2	2	2	0	0	-15	-15	3	3	-3	-3	-1	-1	-1	-1	1	1	3	3	-1	-1	-3	-3	3	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	704	64	0	0	0	0	0	-64 -24ω	-40 24ω	-8ω	8 8ω	0	0	0	0	0	0	8 12ω	-4 12ω	-8	-8	4ω	-4 -4ω	0	0	0	0	0	0	2	-2	0	0	-2ω	2 2ω	2	-ω	1 ω	ω	-1 -ω	-ω	1 ω	-1	-ω	1 ω	0	0
χ_{37}	704	64	0	0	0	0	0	-40 24ω	-64 -24ω	8 8ω	-8ω	0	0	0	0	0	0	-4 -12ω	8 12ω	-8	-8	-4 -4ω	4ω	0	0	0	0	0	0	2	-2	0	0	2 2ω	-2ω	2	1 ω	-ω	-1 -ω	ω	1 ω	-ω	-1	1 ω	-ω	0	0
χ_{38}	880	48	16	0	0	0	0	40 60ω	-20 -60ω	-8 -4ω	-4 4ω	4ω	-4 -4ω	0	0	0	0	10 6ω	4 4ω	6 6ω	-6ω	-2 2ω	-4 -2ω	2 2ω	-2ω	0	0	0	0	-8	0	0	1	1 2ω	-1 -2ω	-5	-2ω	2 2ω	0	0	ω	-1 -ω	0	0	0	0	0
χ_{39}	880	48	16	0	0	0	0	-20 -60ω	40 60ω	-4 4ω	-8 -4ω	-4 -4ω	4ω	0	0	0	0	4 -6ω	10 6ω	-6ω	6 6ω	-4 -2ω	-2 2ω	-2ω	2 2ω	0	0	0	0	-8	0	0	1	-1 -2ω	1 2ω	-5	2 2ω	-2ω	0	0	-1 -ω	ω	0	0	0	0	0
χ_{40}	891	27	-21	3	3	-1	-1	81	81	9	9	-3	-3	-3	-3	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
χ_{41}	891	27	-21	3	3	-1	-1	-81 -81ω	81ω	-9 -9ω	9ω	3 3ω	-3ω	3 3ω	-3ω	1 ω	-ω	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	ω	-1 -ω	0	0
χ_{42}	891	27	-21	3	3	-1	-1	81ω	-81 -81ω	9ω	-9 -9ω	-3ω	3 3ω	-3ω	3 3ω	-ω	1 ω	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1 -ω	ω	0	0
χ_{43}	990	-18	6	-2	-6	-2	0	-45 -45ω	45ω	3 3ω	-3ω	3 3ω	-3ω	-1 -ω	ω	-1 -ω	ω	-18 -18ω	18ω	0	0	6 6ω	-6ω	0	0	2 2ω	-2ω	0	0	-9	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	990	-18	6	-2	-6	-2	0	45ω	-45 -45ω	-3ω	3 3ω	-3ω	3 3ω	ω	-1 -ω	ω	-1 -ω	18ω	-18 -18ω	0	0	-6ω	6 6ω	0	0	-2ω	2 2ω	0	0	-9	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	1024	0	0	0	0	0	0	64	64	0	0	0	0	0	0	0	0	16	16	0	0	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	4	-2	-2	0	0	1	1	-1	-1	-1	1	1
χ_{46}	1215	-81	-9	-9	3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	α	$\bar\alpha$
χ_{47}	1215	-81	-9	-9	3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\bar\alpha$	α

$\omega^3 = 1$

$\alpha\bar\alpha = 3$

$\alpha^2 + \bar\alpha^2 = -5$

$\alpha + \bar\alpha = -1$

§9 Conjugacy classes in PSU(6,2)

The following table calculated using Theorem 6.1 contains a list of the conjugacy classes of the group U(6,2) contained in SU(6,2). Conjugacy classes bracketed together have the same centralizer.

TABLE 3

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$t+1$	6	$2^{15}\cdot 3^{8}\cdot 5\cdot 7\cdot 11$		$2^{15}\cdot 3^{8}\cdot 5\cdot 7\cdot 11$
$t+x$	6			
$t+x+1$	6			
$t+1$ $(t+1)^2$	4 1	$2^{6}\cdot 3^{5}\cdot 5$ 3	$4^{4+\frac{1}{2}}\cdot 1$	$2^{15}\cdot 3^{6}\cdot 5$
$t+x$ $(t+x)^2$	4 1			
$t+x+1$ $(t+x+1)^2$	4 1			
$t+1$ $(t+1)^2$	2 2	$2\cdot 3^{2}$ $2\cdot 3^{2}$	$4^{4+\frac{1}{2}}\cdot 2^{2}$	$2^{14}\cdot 3^{4}$
$t+x$ $(t+x)^2$	2 2			
$t+x+1$ $(t+x+1)^2$	2 2			
$(t+1)^2$	3	$2^{3}\cdot 3^{4}$	$4^{\frac{1}{2}}\cdot 9$	$2^{12}\cdot 3^{4}$
$(t+x)^2$	3			
$(t+x+1)^2$	3			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$t+1$	3	$2^3.3^4$	$4^{3+\frac{1}{2}.2}$	$2^{11}.3^5$
$(t+1)^3$	1	3		
$t+x$	3			
$(t+x)^3$	1			
$t+x+1$	3			
$(t+x+1)^3$	1			
$t+1$	1	3	$4^{1+1+2+\frac{1}{2}.3}$	$2^{11}.3^3$
$(t+1)^2$	1	3		
$(t+1)^3$	1	3		
$t+x$	1			
$(t+x)^2$	1			
$(t+x)^3$	1			
$t+x+1$	1			
$(t+x+1)^2$	1			
$(t+x+1)^3$	1			
x $(t+1)^3$	2	2.3^2	$4^{\frac{1}{2}.2.2^2}$	$2^9.3^2$
$(t+x)^3$	2			
$(t+x+1)^3$	2			
$t+1$	2	2.3^2	$4^{2+\frac{1}{2}.3}$	$2^8.3^3$
$(t+1)^4$	1	3		
$t+x$	2			
$(t+x)^4$	1			
$t+x+1$	2			
$(t+x+1)^4$	1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$(t+1)^2$	1	3		
$(t+1)^4$	1	3	$4^{2+\frac{1}{2}(1+3)}$	$2^8.3^2$
$(t+x)^2$	1			
$(t+x)^4$	1			
$(t+x+1)^2$	1			
$(t+x+1)^4$	1			
$t+1$	1	3		
$(t+1)^5$	1	3	$4^{1+\frac{1}{2}.4}$	$2^6.3^2$
$t+x$	1			
$(t+x)^5$	1			
$t+x+1$	1			
$(t+x+1)^5$	1			
$(t+1)^6$	1	3	$4^{\frac{1}{2}.5}$	$2^5.3$
$(t+x)^6$	1			
$(t+x+1)^6$	1			
$t+1$	4	$2^6.3^5.5$		
$t+x$	1	3		
$t+x+1$	1	3		$2^6.3^7.5$
$t+x$	4			
$t+x+1$	1			
$t+1$	1			
$t+x+1$	4			
$t+1$	1			
$t+x$	1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$(t+1)^2$	1	3		
$t+1$	2	2.3^2		
$t+x$	1	3		
$t+x+1$	1	3	$4^{2+\frac{1}{2}}.1$	$2^6.3^5$
$(t+x)^2$	1			
$t+x$	2			
$t+x+1$	1			
$t+1$	1			
$(t+x+1)^2$	1			
$t+x+1$	2			
$t+1$	1			
$t+x$	1			
$(t+1)^2$	2	2.3^2		
$t+x$	1	3		
$t+x+1$	1	3	$4^{\frac{1}{2}}.2^2$	$2^5.3^4$
$(t+x)^2$	2			
$t+1$	1			
$t+x+1$	1			
$(t+x+1)^2$	2			
$t+1$	1			
$t+x$	1			
$t+1$	1	3		
$(t+1)^3$	1	3		
$t+x$	1	3		
$t+x+1$	1	3	$4^{1+\frac{1}{2}}.2$	$2^4.3^4$
$t+x$	1			
$(t+x)^3$	1			
$t+1$	1			
$t+x+1$	1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$t+x+1$ $(t+x+1)^3$ $t+1$ $t+x$	1 1 1 1			
$(t+1)^4$ $t+x$ $t+x+1$	1 1 1	3 3 3	$4^{\frac{1}{2}}.3$	$2^3.3^3$
$(t+x)^4$ $t+1$ $t+x+1$	1 1 1			
$(t+x+1)^4$ $t+1$ $t+x$	1 1 1			
$t+1$ $t+x$	3 3	$2^3.3^4$ $2^3.3^4$		$2^6.3^8$
$t+1$ $t+x+1$	3 3			
$t+x+1$ $t+x$	3 3			
$t+1$ $(t+1)^2$ $t+x$	1 1 3	3 3 $2^3.3^4$	$4^{1+\frac{1}{2}}$	$2^6.3^6$
$t+1$ $(t+1)^2$ $t+x+1$	1 1 3			
$t+x$ $(t+x)^2$ $t+1$	1 1 3			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$(t+1)^3$ $t+x$	1 3	3 $2^3.3^4$	$4^{\frac{1}{2}}.2$	$2^5.3^5$
$(t+1)^3$ $t+x+1$	1 3			
$(t+x)^3$ $t+1$	1 3			
$(t+x)^3$ $t+x+1$	1 3			
$(t+x+1)^3$ $t+1$	1 3			
$(t+x+1)^3$ $t+x$	1 3			
$(t+1)^3$ $t+x$ $(t+x)^2$	1 1 1	3 3 3	$4^{\frac{1}{2}}.2$ $4^{1+\frac{1}{2}}$	$2^5.3^3$
$(t+1)^3$ $t+x+1$ $(t+x+1)^2$	1 1 1			
$(t+x)^3$ $t+1$ $(t+1)^2$	1 1 1			
$(t+x)^3$ $t+x+1$ $(t+x+1)^2$	1 1 1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$t+x$	1			
$(t+x)^2$	1			
$t+x+1$	3			
$t+x+1$	1			
$(t+x+1)^2$	1			
$t+1$	3			
$t+x+1$	1			
$(t+x+1)^2$	1			
$t+x$	3			
$t+1$	1	3		
$(t+1)^2$	1	3	$4^{1+\frac{1}{2}}$	
$t+x$	1	3		
$(t+x)^2$	1	3	$4^{1+\frac{1}{2}}$	$2^6.3^4$
$t+1$	1			
$(t+1)^2$	1			
$t+x+1$	1			
$(t+x+1)^2$	1			
$t+x$	1			
$(t+x)^2$	1			
$t+x+1$	1			
$(t+x+1)^2$	1			
$(t+1)^3$	1	3	$4^{\frac{1}{2}}.2$	
$(t+x)^3$	1	3	$4^{\frac{1}{2}}.2$	$2^4.3^2$
$(t+1)^3$	1			
$(t+x+1)^3$	1			
$(t+x)^3$	1			
$(t+x+1)^3$	1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$(t+x+1)^3$	1			
$t+1$	1			
$(t+1)^2$	1			
$(t+x+1)^3$	1			
$t+x$	1			
$(t+x)^2$	1			
* $(t+1)^2$	1	3	$4^{\frac{1}{2}}$	
$(t+x)^2$	1	3	$4^{\frac{1}{2}}$	
$(t+x+1)^2$	1	3	$4^{\frac{1}{2}}$	
$(t+1)^2$	1	3	$4^{\frac{1}{2}}$	
$(t+x)^2$	1	3	$4^{\frac{1}{2}}$	
$t+x+1$	2	2.3^2		$2^3.3^4$
$(t+1)^2$	1			
$(t+x+1)^2$	1			
$t+x$	2			
$(t+x)^2$	1			
$(t+x+1)^2$	1			
$t+1$	2			
$(t+1)^2$	1	3		
$t+x$	2	2.3^2		
$t+x+1$	2	2.3^2	$4^{\frac{1}{2}}$	$2^3.3^5$
$(t+x)^2$	1			
$t+1$	2			
$t+x+1$	2			
$(t+x+1)^2$	1			
$t+1$	2			
$t+x$	2			

TABLE 3 (Contd)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
*	t+1 t+x t+x+1	2 2 2	2.3^2 2.3^2 2.3^2		$2^3.3^6$
*	t^3+x t^3+x+1	1 1	3^2 3^2		3^4
	t^3+x t+x+1 $(t+1)^2$	1 1 1	3^2 3 3	$4^{\frac{1}{2}}$	2.3^4
	t^3+x t+1 $(t+x)^2$	1 1 1			
	t^3+x t+x $(t+x+1)^2$	1 1 1			
	t^3+x+1 t+1 $(t+x+1)^2$	1 1 1			
	t^3+x+1 t+x $(t+1)^2$	1 1 1			
	t^3+x+1 t+x+1 $(t+x)^2$	1 1 1			

TABLE 3 (Contd)

el.div. of X φ^μ	$m(\varphi^\mu)$	$A(\varphi^\mu)$	$B(\varphi^\mu)$	$\lvert C_{U(6,2)}(X)\rvert$
t^3+x $t+x+1$ $t+1$	1 1 2	3^2 3 2.3^2		 2.3^5
t^3+x $t+1$ $t+x$	1 1 2			
t^3+x $t+x$ $t+x+1$	1 1 2			
t^3+x+1 $t+x$ $t+1$	1 1 2			
t^3+x+1 $t+x+1$ $t+x$	1 1 2			
t^3+x+1 $t+1$ $t+x+1$	1 1 2			
$\left.\begin{matrix} t^2+xt+1 \\ t^2+(x+1)t+1 \end{matrix}\right\}$ $(t+1)^2$	1 1	3.5 3	 $4^{\frac{1}{2}}$	 $2.3^2.5$
$\left.\begin{matrix} t^2+t+x+1 \\ t^2+(x+1)t+x+1 \end{matrix}\right\}$ $(t+x)^2$	1 1			
$\left.\begin{matrix} t^2+t+x \\ t^2+xt+x \end{matrix}\right\}$ $(t+x+1)^2$	1 1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$\left.\begin{matrix} t^2+xt+1 \\ t^2+(x+1)+1 \end{matrix}\right\}$ $t+1$	1 2	3.5 2.3^2		 $2.3^3.5$
$\left.\begin{matrix} t^2+t+x+1 \\ t^2+(x+1)t+x+1 \end{matrix}\right\}$ $t+x$	1 2			
$\left.\begin{matrix} t^2+t+x \\ t^2+xt+x \end{matrix}\right\}$ $t+x+1$	1 2			
$\left.\begin{matrix} t^2+xt+1 \\ t^2+(x+1)t+1 \end{matrix}\right\}$ $t+x$ $t+x+1$	1 1 1	3.5 3 3		 $3^3.5$
$\left.\begin{matrix} t^2+t+x+1 \\ t^2+(x+1)t+x+1 \end{matrix}\right\}$ $t+1$ $t+x+1$	1 1 1			
$\left.\begin{matrix} t^2+t+x \\ t^2+xt+x \end{matrix}\right\}$ $t+1$ $t+x$	1 1 1			
$\left.\begin{matrix} t^3+t+1 \\ t^3+t^2+1 \end{matrix}\right\}$	1	$3^2.7$		$3^2.7$
$\left.\begin{matrix} t^3+xt+1 \\ t^3+(x+1)t^2+1 \end{matrix}\right\}$	1			
$\left.\begin{matrix} t^3+(x+1)t+1 \\ t^3+xt^2+1 \end{matrix}\right\}$	1			

TABLE 3 (Contd)

el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$A(\varphi^{\mu})$	$B(\varphi^{\mu})$	$\lvert C_{U(6,2)}(X)\rvert$
$t^5+xt^4+t^3+t^2+(x+1)t+1$	1	3.11		
$t+1$	1	3		$3^2.11$
$t^5+(x+1)t^4+t^3+t^2+xt+1$	1			
$t+1$	1			
$t^5+t^4+xt^3+t^2+xt+x$	1			
$t+x+1$	1			
$t^5+t^4+(x+1)t^3+t^2+(x+1)t+x+1$	1			
$t+x$	1			
$t^5+(x+1)t^4+(x+1)t^3+(x+1)t^2+(x+1)t+1$	1			
$t+x+1$	1			
$t^5+xt^4+xt^3+xt^2+xt+x+1$	1			
$t+x$	1			

On restriction to SU(6,2) all conjugacy classes remain unsplit except the 3 sets of 3 each marked by x. These 9 exceptional classes each split into 3 conjugacy classes in SU(6,2). This is because they consist of matrices similar to ones of the form

$$\left[\begin{array}{ccc:ccc} a & 1 & & & & \\ & a & 1 & & & \\ & & a & & & \\ \hdashline & & & b & 1 & \\ & & & & b & 1 \\ & & & & & b \end{array}\right]$$

The determinant of any matrix in the centralizer of a matrix of this type is 1 as $\alpha^3=1$ for all $\alpha \in GF(4)\backslash\{0\}$.

$Z(U(6,2)) = Z(SU(6,2))$ and has order 3. On factorizing out this centre all conjugacy classes except those marked by * fuse to others in the same bracket. (The brackets containing 6 conjugacy classes of $U(6,2)$ yield 2 classes in $PSU(6,2)$.) The classes marked by * give just one conjugacy class with 1/3 of the order of the class in $SU(6,2)$.

Hence $PSU(6,2)$ has 46 conjugacy classes. (See TABLE 5.)

§10 Characters of PSU(6,2)

We determine the characters of PSU(6,2), $\chi_i^{(6)}$ $i=1,\ldots,46$ mainly from characters induced from the subgroup PSU(5,2) of index 672 and the simple subgroup $P\Omega^-(6,3)$ of index 2816 (Lemma 1.3).

We first discuss the 2 rank 3 representations of degrees 672 and 693, τ_{672} and τ_{693}.

The number of points in PG(6,4) $= 1+4+\ldots+4^5 = 1365$.

Now we recall from §8 that:= $l(\underset{\sim}{x}) = 1$ if and only if $\underset{\sim}{x}$ has an odd number of non-zero coordinates, $l(\underset{\sim}{x}) = 0$ if and only if $\underset{\sim}{x}$ has an even number of non-zero coordinates.

Therefore the number of non-isotropic points is $6 + \binom{6}{3}.3^2 + \binom{6}{5}.3^4 = 672$. The number of isotropic points is $\binom{6}{2}.3 + \binom{6}{4}.3^3 + \binom{6}{6}.3^5 = 693$.

As in §8 for PSU(5,2) the character of the representation on 1365 points can be calculated with knowledge of the elementary divisors of the element.

$$\tau_{672} = (\chi_1^{(5)})^{PSU(6,2)} = (1_{PSU(5,2)})^{PSU(6,2)}.$$

Hence we calculate τ_{672} and τ_{693} on all the conjugacy classes of PSU(6,2).

We now apply the rank 3 results from §5.

τ_{672}: $n=672$ $k=176$ $l=495$. $\lambda=40$

$$\mu l = k(k-\lambda-1) \quad \text{(Equation 5.3)}$$

Therefore $\mu = 176.135/495 = 48$.

$$d = (\lambda-\mu)^2 + 4(k-\lambda) = 576 = 24^2.$$

$$\{f_2,f_3\} = \frac{2k + (\lambda-\mu)(k+1) \mp \sqrt{d}(k+1)}{\mp 2\sqrt{d}} \quad \text{(Equation 5.4)}$$

$$= \frac{2.176 + (-8).671 \mp 24.671}{\mp 2.24}$$

$$= \{440,231\}.$$

τ_{693}: $n=693$ $k=180$ $l=512$ $\lambda=51$

$$\mu = 180.128/512 = 45.$$

$$d = 6^2 + 4.135 = 576 = 24^2.$$

$$\{f_2,f_3\} = \frac{2.180 + 6.692 \mp 24.692}{\mp 2.24}$$

$$= \{440,252\}.$$

τ_{672} is non-zero on 31 of the 46 conjugacy classes. Hence the values of these irreducible characters of degrees 231,252 and 440 can be determined on the 31 classes by restricting the permutation characters to PSU(5,2). The remaining 15 conjugacy classes fall into 3 sets of 3 (with the same elementary divisors) and 6 others. The sets of 3 correspond to the 3 conjugacy classes of subgroups isomorphic to $P\Omega^-(6,3)$ and the values of these characters is therefore constant on a given set of 3 conjugacy classes. This information is sufficient to determine the values of the characters of degrees 231,252 and 440 on all the conjugacy classes.

PSU(6,2) contains 3 conjugacy classes of subgroups isomorphic to $P\Omega^-(6,3)$ (Lemma 1.3). These subgroups are of index 1408 in PSU(6,2) and they contain subgroups of index 2 isomorphic to $PS\Omega^-(6,3)$. The character table of

$PS\Omega^-(6,3) \cong PSU(4,9)$ has been calculated by L. Finklestein, a research student at the University of Birmingham, and it is presented as table 4.

$PS\Omega^-(6,3)$ can be embedded in $PSU(6,2)$ in precisely 3 ways:=

conjugacy class number in $PS\Omega^-(6,3)$	conjugacy class number in $PSU(6,2)$
1	1
2	3
4'	7,8,9
4"	11
8	13,14,15
3'	21
3"	16
3"'	35
3""	35
9'	36
9"	36
9"'	39
9""	40
6'	24
6"	18
6"'	33
12	29,30,31
5	42
7'	46
7"	46

The numbers for $PSU(6,2)$ are obtained by counting from left to right across the character table, TABLE 5. The above solution to the fusion problem can either be obtained by group theoretical or character theoretical methods. The easiest method is probably by restricting

TABLE 4

CHARACTER TABLE OF $PSU(4,9) \simeq PS\Omega^-(6,3)$ OF ORDER 3,265,920

$O(x)$	1	2	4'	4''	8	3'	3''	3'''	3''''	9'	9''	9'''	9''''	6'	6''	6'''	12	5	7'	7''
$\lvert C_{PS\Omega^-(6,3)}(x)\rvert$	$2^7 3^6 5.7$	$2^7.3^2$	$2^5.3$	2^4	2^3	$2^3 3^6$	$2^2 3^5$	$2^2 3^5$	3^4	3^3	3^3	3^3	3^3	$2^3 3^2$	$2^2 3^2$	$2^2 3^2$	$2^2 3$	5	7	7
	3265920	1152	96	16	8	5832	972	972	81	27	27	27	27	72	36	36	12	5	7	7
$\lvert ccl(x)\rvert$	1	$3^4 5.7$	$2^2.3^5 5.7$	$2^3 3^6 5.7$	$2^4.3^6 5.7$	$2^4.5.7$	$2^5 3 5.7$	$2^5 3 5.7$	$2^7 3^2 5.7$	$2^7 3^3 5.7$	$2^7.3^3 5.7$	$2^7.3^3 5.7$	$2^7.3^3 5.7$	$2^4 3^4 5.7$	$2^5.3^4 5.7$	$2^5.3^4 5.7$	$2^5.3^5 5.7$	$2^7.3^6.7$	$2^7.3^6 5$	$2^7.3^6 5$
	1	2835	34020	204120	408240	560	3360	3360	40320	120960	120960	120960	120960	45360	90720	90720	272160	653184	466560	466560
rep x																				
α_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
α_2	21	5	1	1	-1	-6	3	3	3	0	0	0	0	2	-1	-1	-2	1	0	0
α_3	35	3	3	-1	-1	8	8	-1	-1	-1	-1	2	2	0	0	3	0	0	0	0
α_4	35	3	3	-1	-1	8	-1	8	-1	2	2	-1	-1	0	3	0	0	0	0	0
α_5	90	10	-2	2	0	9	9	9	0	0	0	0	0	1	1	1	1	0	-1	-1
α_6	140	12	4	0	0	5	-4	-4	5	-1	-1	-1	-1	3	0	0	1	0	0	0
α_7	189	-3	5	1	1	27	0	0	0	0	0	0	0	3	0	0	-1	-1	0	0
α_8	210	2	-2	-2	0	21	3	3	3	0	0	0	0	5	-1	-1	1	0	0	0
α_9	280	-8	0	0	0	10	1	10	1	$1+3\bar\rho$	$1+3\rho$	1	1	-2	1	-2	0	0	0	0
α_{10}	280	-8	0	0	0	10	1	10	1	$1+3\rho$	$1+3\bar\rho$	1	1	-2	1	-2	0	0	0	0
α_{11}	280	-8	0	0	0	10	10	1	1	1	1	$1+3\bar\rho$	$1+3\rho$	-2	-2	1	0	0	0	0
α_{12}	280	-8	0	0	0	10	10	1	1	1	1	$1+3\rho$	$1+3\bar\rho$	-2	-2	1	0	0	0	0
α_{13}	315	11	-1	-1	1	-9	18	-9	0	0	0	0	0	-1	2	-1	-1	0	0	0
α_{14}	315	11	-1	-1	1	-9	-9	18	0	0	0	0	0	-1	-1	2	-1	0	0	0
α_{15}	420	4	4	0	0	-39	6	6	-3	0	0	0	0	1	-2	-2	1	0	0	0
α_{16}	560	-16	0	0	0	-34	2	2	2	-1	-1	-1	-1	2	2	2	0	0	0	0
α_{17}	640	0	0	0	0	-8	-8	-8	1	1	1	1	1	0	0	0	0	0	σ	$\bar\sigma$
α_{18}	640	0	0	0	0	-8	-8	-8	1	1	1	1	1	0	0	0	0	0	$\bar\sigma$	σ
α_{19}	729	9	-3	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1
α_{20}	896	0	0	0	0	32	-4	-4	-4	-1	-1	-1	-1	0	0	0	0	1	0	0

characters from PSU(6,2) to the subgroup. The character of degree 231 on restriction to $PS\Omega^-(6,3)$ clearly splits as $\alpha_2 + \alpha_{10}$. As $\alpha_2(2) + \alpha_{10}(2) = 7$ the involutions are elements of T (see §1 for defn. of T). Therefore 2 fuses to 3. Similar arguments give the other fusions.

The induced character $(1_{PS\Omega^-(6,3)})^{PSU(6,2)}$ is the sum of 5 irreducible characters including $1_{PSU(6,2)}$ and the character of degree 252 already determined. The method of determining the other 3 characters which is fully described below is used in several places in this thesis and in future will simply be loosely referred to as "restricting and splitting".

We aim to solve the equation 1+252+b+c+d for the degrees of the other 3 irreducible characters. However we know that the set of 5 degrees partition into 2 subsets each with sum 1408.

We restrict the character to PSU(5,2). The character splits as $\chi_1^{(5)} + \chi_9^{(5)} + \chi_{10}^{(5)} + \chi_{16}^{(5)} + \chi_3^{(5)} + \chi_4^{(5)} + \chi_6^{(5)} + \chi_{35}^{(5)} + \chi_{28}^{(5)} + \chi_{12}^{(5)} + \chi_{13}^{(5)} + \chi_{18}^{(5)} + \chi_{33}^{(5)} + \chi_{34}^{(5)}$ which expressed as a sum of degrees is 1+252+22+55+176+220+440 +660+990 collecting those characters which make up the 252 together.

990+660>1408, 990+440>1408 ∴ 1408=660+ 440+... 1408-(660+220+440)=86 which cannot be expressed as a sum of the remaining degrees. $\therefore$ $1408=990+220+\ldots$

$1408<990+220+252$. $1408-(990+220)=198$.

$$1408-(660+440+252)=56.$$

$\therefore$ $1408=1+252+660+440+55$

and $1408=990+220+176+22$

Now either $1408=1+252+b+c$ and $1408=d$,

or $1408=1+252+b$ and $1408=c+d$.

We now restrict the character to $PS\Omega^{-}(6,3)$ and we get

$$2816=5.1+3.21+189+729+4.90+3.140+2.315+420.$$

In the first of the 2 cases we would have a character of PSU(6,2) of degree either 55,440 or 660. Considering congruences mod 7 we see none of these 3 integers can be expressed as sum of numbers in the above equation.

In the second case we already have b=1155. There are just 2 partitions of $\{990,220,176,22\}$ for which the sums both divide $2^{15}.3^{6}.5.7.11$.

$1408=22+1386$ or $1408=176+1232$.

176 cannot be expressed as a sum of numbers in the above equation.

Therefore PSU(6,2) has a character of degree 22 and one of degree 1386. The values of these 3 characters (degree 22,1155 and 1386) can now easily be determined using the 1 conjugacy class of subgroups isomorphic to PSU(5,2) and the 3 conjugacy classes of subgroups isomorphic to $PS\Omega^{-}(6,3)$. There are 3 conjugate characters

TABLE 5

CHARACTER TABLE OF PSU(6,2)

ORDER = 9,196,830,720 $= 2^{15}.3^6.5.7.11$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
ORDER (x)	1	2	2	2	4	4	4	4	4	4	4	8	8	8	8	3	6	6	12	12	3	6	6	6	12	12	12	12	12	12	12	6	6	6	3	9	18	18	9	9	10	5	15	11	11	7
$\lvert C_{PSU(6,2)}(x)\rvert$	$2^{15}3^6 5\,7\,11$	$2^{15}3^4 5$	$2^{14}3^2$	$2^{12}3^2$	$2^{11}3^3$	$2^{11}3$	$2^9 3$	$2^9 3$	$2^9 3$	$2^8 3$	2^8	2^6	2^5	2^5	2^5	$2^6 3^5 5$	$2^6 3^3$	$2^5 3^2$	$2^4 3^2$	$2^3 3$	$2^6 3^6$	$2^6 3^4$	$2^6 3^4$	$2^6 3^2$	$2^5 3^3$	$2^5 3^3$	$2^5 3$	$2^5 3$	$2^4 3$	$2^4 3$	$2^4 3$	$2^3 3^2$	$2^3 3^2$	$2^3 3^3$	$2^3 3^5$	3^3	$2\,3^2$	$2\,3^2$	$2\,3^3$	$2\,3^3$	$2\,5$	$2\,3\,5$	$3\,5$	11	11	7
$\lvert cl(x)\rvert$	1	693	62370	249480	166320	1496880	5987520	5987520	5987520	11975040	35925120	143700480	287400960	287400960	287400960	118272	5322240	31933440	63866880	383201280	197120	1774080	1774080	15966720	10644480	10644480	95800320	95800320	191600640	191600640	191600640	127733760	127733760	42577920	4730880	340623360	510935040	510935040	170311680	170311680	919683072	306561024	613122048	836075520	836075520	1313832960
[illegible] (x)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	22	-10	6	-2	6	-2	2	2	2	-2	2	2	0	0	0	4	-4	0	0	-2	-5	-1	-1	3	-3	-3	1	1	-1	-1	-1	-2	0	2	4	1	-1	-1	1	1	0	2	-1	0	0	1
χ_3	231	34	7	-9	23	7	-1	-1	-1	-1	-1	3	-1	-1	-1	6	6	-2	2	2	15	3	3	7	5	5	1	1	-1	-1	-1	0	-2	0	6	0	0	0	0	0	-1	1	1	0	0	0
χ_4	252	60	28	12	12	-4	4	4	4	4	4	0	0	0	0	9	9	1	-3	1	9	-3	-3	1	3	3	-1	-1	1	1	1	3	1	3	9	0	0	0	0	0	0	2	-1	-1	-1	0
χ_5	385	-95	17	-7	1	9	5	5	5	-7	-3	1	-1	-1	-1	25	1	5	1	-1	7	-5	-5	-1	1	1	-3	-3	-1	-1	-1	2	2	-2	-2	1	1	1	1	1	0	0	0	0	0	0
χ_6	440	120	24	8	24	8	0	0	0	8	0	0	0	0	0	35	3	3	3	-1	8	12	12	0	6	6	2	2	0	0	0	-1	3	3	-1	-1	0	0	2	2	0	0	0	0	0	-1
χ_7	560	-80	-16	16	16	-16	0	0	0	0	0	0	0	0	0	20	4	-4	4	0	20	-8	-8	-4	-2	-2	2	2	0	0	0	-2	2	-2	2	2	1	1	-1	-1	0	0	0	-1	-1	0
χ_8	616	-24	40	8	8	8	8	8	8	0	0	0	0	0	0	-14	-6	-2	2	0	-5	3	3	-5	-1	-1	-1	-1	-1	-1	-1	-1	1	3	13	-2	0	0	-2	-2	1	1	1	0	0	0
χ_9	770	-30	-14	10	34	-6	-2	-2	-2	2	-2	2	0	0	0	5	-3	1	1	-1	-13	$3+12\omega$	$-9-12\omega$	7	$-5-6\omega$	$1+6\omega$	$-1-2\omega$	$1+2\omega$	1	1	1	1	-1	-3	5	-1	$1+2\omega$	$-1-2\omega$	-1	-1	0	0	0	0	0	0
χ_{10}	770	-30	-14	10	34	-6	-2	-2	-2	2	-2	2	0	0	0	5	-3	1	1	-1	-13	$-9-12\omega$	$3+12\omega$	7	$1+6\omega$	$-5-6\omega$	$1+2\omega$	$-1-2\omega$	1	1	1	1	1	-3	5	-1	$1+2\omega$	$-1-2\omega$	-1	-1	0	0	0	0	0	0
χ_{11}	1155	195	35	19	-13	3	11	-5	-5	3	3	-1	3	-1	-1	30	6	2	2	0	-6	6	6	2	-4	-4	0	0	2	-2	-2	1	-1	-3	3	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	1155	195	35	19	-13	3	-5	11	-5	3	3	-1	-1	3	-1	30	6	2	2	0	-6	6	6	2	-4	-4	0	0	-2	2	-2	1	-1	-3	3	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	1155	195	35	19	-13	3	-5	-5	11	3	3	-1	-1	-1	3	30	6	2	2	0	-6	6	6	2	-4	-4	0	0	-2	-2	2	1	-1	-3	3	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	1386	-246	58	-30	-6	2	-2	-2	-2	-6	6	-2	0	0	0	36	-12	4	0	0	9	-3	-3	1	3	3	-1	-1	1	1	1	-3	1	-3	9	0	0	0	0	0	-1	1	1	0	0	0
χ_{15}	1540	260	4	-28	-12	-12	4	4	4	4	-4	0	0	0	0	55	-1	-5	3	1	1	17	17	1	-3	-3	-3	-3	1	1	1	-1	1	-1	1	1	-1	-1	1	1	0	0	0	0	0	0
χ_{16}	3080	-440	40	-8	40	-8	0	0	0	-8	0	0	0	0	0	65	1	1	1	1	2	$-2-12\omega$	$-14-12\omega$	-2	$-8-6\omega$	$-2+6\omega$	-2ω	$2+2\omega$	0	0	0	1	1	1	-7	-1	$-1-\omega$	ω	$-1-3\omega$	$2+3\omega$	0	0	0	0	0	0
χ_{17}	3080	-440	40	-8	40	-8	0	0	0	-8	0	0	0	0	0	65	1	1	1	1	2	$-14-12\omega$	$-2+12\omega$	-2	$-2+6\omega$	$-8-6\omega$	$2+2\omega$	-2ω	0	0	0	1	1	1	-7	-1	ω	$-1-\omega$	$2+3\omega$	$-1-3\omega$	0	0	0	0	0	0
χ_{18}	3520	-320	64	0	64	0	0	0	0	0	0	0	0	0	0	10	-14	-2	-2	0	-44	4	4	4	-8	-8	0	0	0	0	0	0	-2	4	10	1	1	1	1	1	0	0	0	0	0	0
χ_{19}	4620	-180	44	-36	28	-20	4	4	4	-4	-4	0	0	0	0	-15	9	5	1	-1	57	9	9	-7	1	1	1	1	1	1	1	0	-4	0	12	0	0	0	0	0	0	0	0	0	0	-1
χ_{20}	4928	320	64	64	0	0	0	0	0	0	0	0	0	0	0	-4	-4	4	0	0	68	-4	-4	4	0	0	0	0	0	0	0	1	1	5	5	2	-1	-1	-1	-1	0	-2	1	0	0	0
χ_{21}	5544	-24	-56	24	72	24	0	0	0	-8	0	0	0	0	0	9	9	1	-3	1	36	12	12	4	0	0	0	0	0	0	0	-3	1	-3	9	0	0	0	0	0	1	-1	-1	0	0	0
χ_{22}	6160	400	-48	-16	48	16	0	0	0	0	0	0	0	0	0	40	-8	0	0	0	-50	$-2-12\omega$	$10+12\omega$	-6	$6+6\omega$	-6ω	$2+2\omega$	-2ω	0	0	0	2	0	-2	4	1	ω	$-1-\omega$	$-2-3\omega$	$1+3\omega$	0	0	0	0	0	0
χ_{23}	6160	400	-48	-16	48	16	0	0	0	0	0	0	0	0	0	40	-8	0	0	0	-50	$10+12\omega$	$-2-12\omega$	-6	-6ω	$6+6\omega$	-2ω	$2+2\omega$	0	0	0	2	0	-2	4	1	$-1-\omega$	ω	$1+3\omega$	$-2-3\omega$	0	0	0	0	0	0
χ_{24}	6930	690	98	42	18	-6	6	6	6	10	-2	2	0	0	0	45	-3	5	-3	1	-36	-12	-12	-4	0	0	0	0	0	0	0	-3	-1	-3	-9	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	8064	384	128	0	0	0	0	0	0	0	0	0	0	0	0	-36	12	-4	0	0	-36	-12	-12	-4	0	0	0	0	0	0	0	0	2	0	18	0	0	0	0	0	-1	-1	-1	1	1	0
χ_{26}	9240	-360	88	-8	-8	-8	24	-8	-8	0	0	0	0	0	0	-30	-6	-2	-2	0	33	9	9	1	1	1	1	1	-3	1	1	1	1	-3	-3	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	9240	-360	88	-8	-8	-8	-8	24	-8	0	0	0	0	0	0	-30	-6	-2	-2	0	33	9	9	1	1	1	1	1	1	-3	1	1	1	-3	-3	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	9240	-360	88	-8	-8	-8	-8	-8	24	0	0	0	0	0	0	-30	-6	-2	-2	0	33	9	9	1	1	1	1	1	1	1	-3	1	1	-3	-3	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	10395	315	-21	-45	75	-13	-1	-1	-1	3	-1	-1	1	1	1	0	0	0	0	0	27	-9	-9	3	$-3-12\omega$	$9+12\omega$	$1+4\omega$	$-3-4\omega$	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	10395	315	-21	-45	75	-13	-1	-1	-1	3	-1	-1	1	1	1	0	0	0	0	0	27	-9	-9	3	$9+12\omega$	$-3-12\omega$	$-3-4\omega$	$1+4\omega$	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	10395	315	-21	-45	-21	19	15	-1	-1	3	-1	-1	-3	1	1	0	0	0	0	0	27	-9	-9	3	-3	-3	1	1	3	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	10395	315	-21	-45	-21	19	-1	15	-1	3	-1	-1	1	-3	1	0	0	0	0	0	27	-9	-9	3	-3	-3	1	1	-1	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	10395	315	-21	-45	-21	19	-1	-1	15	3	-1	-1	1	1	-3	0	0	0	0	0	27	-9	-9	3	-3	-3	1	1	-1	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	11264	1024	0	0	0	0	0	0	0	0	0	0	0	0	0	104	-8	0	0	0	32	16	16	0	0	0	0	0	0	0	0	0	0	4	-4	-1	+1	1	-1	-1	-1	-1	-1	0	0	1
χ_{35}	13860	420	100	36	84	20	4	4	4	-4	4	0	0	0	0	-45	3	-5	3	-1	9	-3	-3	1	3	3	-1	-1	1	1	1	0	-2	0	-18	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	14784	-1344	64	0	-64	0	0	0	0	0	0	0	0	0	0	114	-6	-2	2	0	-12	-12	-12	4	8	8	0	0	0	0	0	0	-2	0	6	0	0	0	0	0	1	-1	-1	0	0	0
χ_{37}	18711	-1161	-57	63	-9	15	3	3	3	-1	-5	-1	1	1	1	81	9	-3	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0
χ_{38}	18711	1431	87	-9	-9	-9	-9	-9	-9	-1	-1	-1	-1	-1	-1	81	9	-3	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
χ_{39}	20790	-810	6	-18	6	14	-6	-6	-6	6	2	2	0	0	0	0	0	0	0	0	54	$18+36\omega$	$-18-36\omega$	-6	6ω	$-6-6\omega$	2ω	$-2-2\omega$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	20790	-810	6	-18	6	14	-6	-6	-6	6	2	2	0	0	0	0	0	0	0	0	54	$-18-36\omega$	$18+36\omega$	-6	$-6-6\omega$	6ω	$-2-2\omega$	2ω	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	24640	-960	64	-64	0	0	0	0	0	0	0	0	0	0	0	-20	12	4	0	0	-92	12	12	4	0	0	0	0	0	0	0	-1	1	3	-11	1	0	0	-2	-2	0	0	0	0	0	0
χ_{42}	25515	-405	-117	27	27	-21	3	3	3	3	3	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha$	$-\bar{\alpha}$	0
χ_{43}	25515	-405	-117	27	27	-21	3	3	3	3	3	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\bar{\alpha}$	$-\alpha$	0
χ_{44}	32768	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-64	0	0	0	0	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	8	-1	0	0	2	2	0	-2	1	-1	-1	1
χ_{45}	37422	270	30	54	-18	6	-6	-6	-6	-2	-6	-2	0	0	0	-81	-9	3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	0	0	0
χ_{46}	40095	1215	-81	-9	-81	-9	3	3	3	-9	3	3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1

$\omega^3 = 1$

$\alpha\bar{\alpha} = 3$

$\alpha^2 + \bar{\alpha}^2 = -5$

$\alpha + \bar{\alpha} = -1$

§11 Conjugacy classes and character tables of $PS\Omega^+(6,3)$ and $PS\Omega(7,3)$

The conjugacy classes in orthogonal groups can be classfied using methods given by G.E. Wall [11]. The situation is more complicated than in the unitary groups and more notation is needed.

Let $X \varepsilon O(n,q)$ and suppose its elementary divisors are :=

$(t-1)^i$ with multiplicity μ_i.

$(t+1)^i$ with multiplicity ν_i.

φ^μ with multiplicity m_μ. $\varphi \neq t+1, t-1$.

The elementary divisors $(t+1)^{2i+1}$ and $(t-1)^{2i+1}$ are called exceptional.

As described in §4 there is associated with X a sesquilinear form $g_X = [x,y]$ on $W_X = \mathrm{Image}(I-X)$.

<u>Lemma 11.1</u> $X|W_X = -P^{-1}$

Now Wall [11] shows that :=

(a) Two elements $X,Y \varepsilon O(n,q)$ are conjugate if and only if their forms are equivalent.

(b) Equivalent forms have similar multipliers.

(c) Two forms with the same multiplier are not always equivalent. We express a given form as a standard form times a representative.

(i) Decompose P as a direct sum of multipliers, $\oplus P_i$; suppose $P_i = P_j$ if P_i is equivalent to P_j. Construct f_i a standard form for P_i. Then f the standard form for P

$$f = \begin{bmatrix} f_1 & & & & & \\ & f_1 & & & & \\ & & \ddots & & & \\ & & & f_1 & & \\ & & & & f_2 & \\ & & & & & \ddots \end{bmatrix}$$

(ii) Obtain one representative of each equivalence class of forms with multiplier P by multiplying f by the matrix $Q = \oplus Q^{ii}$ where $Q^{ii} = I$ if i is not exceptional, and Q^{ii} is a block matrix whose entries are in $k[I_{\deg f_i}]$, where $k = GF(q)$, and which satisfies $Q^{ii} = (Q^{ii})^T$, if i is exceptional. These forms are in 1-1 correspondence with orthogonal forms.

<u>Definition</u> The Hermitian invariant $\psi_i(X)$ = type of Q^{ii} as the matrix of an orthogonal form over the field $k[I_{\deg f_i}]$, i.e. takes the value $\underline{0}$, $\underline{1}$, $\underline{\omega}$ or $\underline{\delta}$.

Let $\psi^{+}_{2i-1}(X)$ and $\psi^{-}_{2i+1}(X)$ be the Hermitian invariants associated with the elementary divisors $(t+1)^{2i-1}$ and $(t-1)^{2i+1}$ of X respectively. ψ^{+}_{2i-1}, ψ^{-}_{2i+1} are bilinear forms over GF(q). Define ψ^{-}_{1} as the type of the core of F on W.

<u>Theorem 11.2 (Wall [11])</u>

(i) X is similar to an element of some orthogonal group O(n,q), if and only if,

(a) $X \sim X^{*-1}$,

(b) Each elementary divisor $(t \pm 1)^{2k}$ of X has even multiplicity.

(i)' Let n be even and suppose that (a) and (b) in (i) are satisfied. If any elementary divisor $(t \pm 1)^{2k+1}$ has positive multiplicity, X is similar to an element of $O^+(n,q)$ and also to an element $O^-(n,q)$. If every such elementary divisor has multiplicity zero, X is similar to an element of $O^+(n,q)$ [$O^-(n,q)$] if and only if $\sum_{\varphi,\mu} \mu m(\varphi^\mu) \equiv 0$ [1] (mod 2).

(ii) 2 elements X, Y of $O(n,q)$ are conjugate in $O(n,q)$ if and only if

(a) $X \sim Y$,

(b) $\psi^+_{2i+1}(X) \approx \psi^+_{2i+1}(Y)$ and $\psi^-_{2i+1}(X) \approx \psi^-_{2i+1}(Y)$ $(i = 0,1,\ldots)$.

(iii) Let k_n^+, k_n^- denote the numbers of conjugacy classes in $O^+(n,q)$, $O^-(n,q)$ respectively. Then

$$\sum_{n=0}^{\infty} (k_n^+ + k_n^-)\, t^n = \prod_{\lambda=1}^{\infty} \left[\frac{(1+t^{2\lambda-1})^4}{1-qt^{2\lambda}}\right]$$

$$\sum_{n=0}^{\infty} (k_n^+ - k_n^-)\, t^n = \prod_{\lambda=1}^{\infty} \left[\frac{1-t^{4\lambda-2}}{1-qt^{4\lambda}}\right]$$

(iv) Let $X \in O(n,q)$. Then the order of $C_{O(n,q)}(X)$ is $\prod_\varphi B(\varphi)$ where

$$B(\varphi) = Q^{\sum_{\mu<\nu} \mu m_\mu m_\nu + \sum_\mu \frac{1}{2}(\mu-1) m_\mu^2} \prod_\mu A(\varphi^\mu)$$

where $Q = q^{|\varphi|}$, $m_\mu = m(\varphi^\mu)$ and

$$A(\varphi^\mu) = \begin{cases} |U(m_\mu, Q)| & (\varphi = \tilde{\varphi}) \quad (\varphi \neq t \pm 1) \\ |GL(m_\mu, Q)|^{\frac{1}{2}} & (\varphi \neq \tilde{\varphi}) \quad (\varphi \neq t \pm 1) \\ |O(m_\mu, Q)| & (\varphi = t \pm 1) \quad (\mu \text{ odd}) \\ q^{-\frac{1}{2} m_\mu} |Sp(m_\mu, Q)| & (\varphi = t \pm 1) \quad (\mu \text{ even}) \end{cases}$$

Here $O(m_\mu, Q)$ is the orthogonal group of the corresponding Hermitian invariant $\psi_\mu^\pm(X)$. 44

To determine all the conjugacy classes with given elementary divisors $\underset{\sim}{\delta}$ or $\underset{\sim}{1}$ is assigned to an elementary divisor $(t\pm 1)^n$, n odd if its multiplicity is odd and $\underset{\sim}{0}$ or $\underset{\sim}{\omega}$ is assigned to the same elementary divisor if its multiplicity is even. The assignments are made such that the total sum $\sum_\mu \psi_\mu^\pm(X) + \sum_{\substack{\mu \\ \varphi \neq t\pm 1}} m_\mu \cdot \underset{\sim}{\omega} = \begin{cases} \underset{\sim}{1} \\ \underset{\sim}{0} \\ \underset{\sim}{\omega} \end{cases}$ according as the group is $\begin{cases} O(2n+1,q) \\ O^+(2n,q) \\ O^-(2n,q). \end{cases}$

When q=3 the addition table for Witt invariants is

+	0	1	ω	δ
0	0	1	ω	δ
1	1	ω	δ	0
ω	ω	δ	0	1
δ	δ	0	1	ω

As an example the conjugacy classes in O(7,3) with elementary divisors $(t-1)^3, t-1, t+1, t^2+1$ are 4 in number with Witt invariants

ψ_1^-	ψ_3^-	ψ_1^+
δ	1	δ
δ	δ	1
1	δ	δ
1	1	1

The orthogonal groups occuring as subgroups of M(22) that we deal with in this chapter are all orthogonal groups over GF(3) and subgroups of $PS\Omega(7,3)$.

The irreducible polynomials over GF(3) of degree $\leqslant 7$ are as follows.

<u>SELF-CONJUGATE</u>

degree 1 $t+1, t-1$;

degree 2 t^2+1;

degree 3 none

degree 4 $t^4+t^3+t^2+t+1,\ t^4-t^3+t^2-t+1$;

degree 5 none

degree 6 $t^6+t^5+t^4+t^3+t^2+t+1,\ t^6-t^5+t^4-t^3+t^2-t+1$,

$t^6+t^5+t^3+t+1,\ t^6-t^5-t^3-t+1$.

<u>CONJUGATE PAIRS</u>

degree 2 t^2+t-1, t^2-t-1;

degree 3 t^3-t+1, t^3-t^2+1;

t^3-t-1, t^3+t^2-1;

t^3+t^2-t+1, t^3-t^2+t+1;

t^3+t^2+t-1, t^3-t^2-t-1.

§12 Conjugacy classes in $PS\Omega^+(6,3)$

We first apply Theorem 11.2 to find the conjugacy classes in $O^+(6,3)$.

For convenience we list here the order of all general linear, symplectic, unitary and orthogonal groups needed for finding the orders of the centralizers of elements in $O^+(6,3)$ and $O(7,3)$.

group	order
$O(1,3)$	2
$O^+(2,3)$	2^2
$O^-(2,3)$	2^3
$O(3,3)$	$2^4.3$
$O^+(4,3)$	$2^7.3^2$
$O^-(4,3)$	$2^5.3^2.5$
$O(5,3)$	$2^8.3^4.5$
$O^+(6,3)$	$2^9.3^6.5.13$
$O^-(6,3)$	$2^{10}.3^6.5.7$
$O(7,3)$	$2^{11}.3^9.5.7.13$
$U(1,9)$	2^2
$U(1,81)$	2.5
$U(1,729)$	$2^2.7$
$U(2,9)$	$2^5.3$
$U(3,9)$	$2^7.3^3.7$
$GL(1,9)$	2^3
$GL(1,27)$	2.13
$Sp(2,3)$	$2^3.3$

TABLE 6 Conjugacy classes in $O^+(6,3)$

	el.div. of X φ^μ	$m(\varphi^\mu)$	$\psi_1^+\ \psi_3^+\ \psi_5^+$	$\psi_1^-\ \psi_3^-\ \psi_5^-$	$A(\varphi^\mu)$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
†*	$t-1$	6		0	$2^9.3^6.5.13$		$2^9.3^6.5.13$
†*	$t-1$ $(t-1)^2$	2 2		0	2^2 $3^{-1}2^3.3$	$3^{4+\frac{1}{2}}.2^2$	$2^5.3^6$
†*	$t-1$ $(t-1)^3$	3 1		δ 1	$2^4.3$ 2	$3^{3+\frac{1}{2}}.2$	$2^5.3^5$
†*	$t-1$ $(t-1)^3$	3 1		1 δ	$2^4.3$ 2	$3^{3+\frac{1}{2}}.2$	$2^5.3^5$
†*	$(t-1)^3$	2		0	2^2	$3^{\frac{1}{2}}.2.2^2$	$2^2.3^4$
†*	$t-1$ $(t-1)^5$	1 1		δ 1	2 2	$3^{1+\frac{1}{2}}.4$	$2^2.3^3$
†*	$t-1$ $(t-1)^5$	1 1		1 δ	2 2	$3^{1+\frac{1}{2}}.4$	$2^2.3^3$
*	$t+1$	6	0		$2^9.3^6.5.13$		$2^9.3^6.5.13$
*	$t+1$ $(t+1)^2$	2 2	0		2^2 $3^{-1}2^3.3$	$3^{4+\frac{1}{2}}.2^2$	$2^5.3^6$
*	$t+1$ $(t+1)^3$	3 1	δ 1		$2^4.3$ 2	$3^{3+\frac{1}{2}}.2$	$2^5.3^5$
*	$t+1$ $(t+1)^3$	3 1	1 δ		$2^4.3$ 2	$3^{3+\frac{1}{2}}.2$	$2^5.3^5$
*	$(t+1)^3$	2	0		2^2	$3^{\frac{1}{2}}.2.2^2$	$2^2.3^4$
*	$t+1$ $(t+1)^5$	1 1	δ		2 2	$3^{1+\frac{1}{2}}.4$	$2^2.3^3$

TABLE 6 (Contd)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	ψ_1^+	ψ_3^+	ψ_5^+	ψ_1^-	ψ_3^-	ψ_5^-	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
†*	$(t-1)^5$	1							2		
	$t+1$	1	δ					1	2	$3^{\frac{1}{2}}.4$	$2^2.3^2$
	$(t-1)^5$	1							2		
	$t+1$	1	1					δ	2	$3^{\frac{1}{2}}.4$	$2^2.3^2$
†	$t+1$	5							$2^8.3^4.5$		
	$t-1$	1	δ			1			2		$2^9.3^4.5$
	$t+1$	5							$2^8.3^4.5$		
	$t-1$	1	1			δ			2		$2^9.3^4.5$
†	$(t+1)^2$	2							$3^{-1}2^3.3$		
	$t+1$	1							2		
	$t-1$	1	δ			1			2	3^{2+2}	$2^5.3^4$
	$(t+1)^2$	2							$3^{-1}2^3.3$		
	$t+1$	1							2		
	$t-1$	1	1			δ			2	3^{2+2}	$2^5.3^4$
	$(t+1)^3$	1							2		
	$t+1$	2							2^2		
	$t-1$	1	0	1		δ			2	$3^{2+\frac{1}{2}}.2$	$2^4.3^3$
	$(t+1)^3$	1							2		
	$t+1$	2							2^3		
	$t-1$	1	ω	δ		δ			2	$3^{2+\frac{1}{2}}.2$	$2^5.3^3$
†	$(t+1)^3$	1							2		
	$t+1$	2							2^2		
	$t-1$	1	0	δ		1			2	$3^{2+\frac{1}{2}}.2$	$2^4.3^3$
†	$(t+1)^3$	1							2		
	$t+1$	2							2^3		
	$t-1$	1	ω	1		1			2	$3^{2+\frac{1}{2}}.2$	$2^5.3^3$

TABLE 6 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	ψ_1^+	ψ_3^+	ψ_5^+	ψ_1^-	ψ_3^-	ψ_5^-	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
*	$t+1$	1							2		
	$(t+1)^5$	1	1		δ				2	$3^{1+\frac{1}{2}}.4$	$2^2.3^3$
†	$t-1$	5							$2^8.3^4.5$		
	$t+1$	1	δ			1			2		$2^9.3^4.5$
	$t-1$	5							$2^8.3^4.5$		
	$t+1$	1	1			δ			2		$2^9.3^4.5$
†	$t-1$	1							2		
	$(t-1)^2$	2							$3^{-1}2^3.3$		
	$t+1$	1	δ			1			2	3^{2+2}	$2^5.3^4$
	$t-1$	1							2		
	$(t-1)^2$	2							$3^{-1}2^3.3$		
	$t+1$	1	1			δ			2	3^{2+2}	$2^5.3^4$
†	$(t-1)^3$	1							2		
	$t-1$	2							2^2		
	$t+1$	1	δ			0	1		2	$3^{2+\frac{1}{2}}.2$	$2^4.3^3$
†	$(t-1)^3$	1							2		
	$t-1$	2							2^3		
	$t+1$	1	δ			ω	δ		2	$3^{2+\frac{1}{2}}.2$	$2^5.3^3$
	$(t-1)^3$	1							2		
	$t-1$	2							2^2		
	$t+1$	1	1			0	δ		2	$3^{2+\frac{1}{2}}.2$	$2^4.3^3$
	$(t-1)^3$	1							2		
	$t-1$	2							2^3		
	$t+1$	1	1			ω	1		2	$3^{2+\frac{1}{2}}.2$	$2^5.3^3$

TABLE 6 (Contd.)

	el.div. of X φ^μ	$m(\varphi^\mu)$	ψ_1^+	ψ_3^+	ψ_5^+	ψ_1^-	ψ_3^-	ψ_5^-	$A(\varphi^\mu)$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
	$(t+1)^5$	1							2		
	$t-1$	1			δ	1			2	$3^{\frac{1}{2}}.4$	$2^2.3^2$
†	$(t+1)^5$	1							2		
	$t-1$	1			1	δ			2	$3^{\frac{1}{2}}.4$	$2^2.3^2$
*	$t-1$	4							$2^7.3^2$		
	$t+1$	2	0			0			2^2		$2^9.3^2$
†*	$t-1$	4							$2^5.3^2.5$		
	$t+1$	2	ω			ω			2^3		$2^8.3^2.5$
*	$(t-1)^2$	2							$3^{-1}2^3.3$		
	$t+1$	2	0						2^2	$3^{\frac{1}{2}}.2^2$	$2^5.3^2$
*	$(t-1)^3$	1							2		
	$t-1$	1							2		
	$t+1$	2	0			δ	1		2^2	$3^{1+\frac{1}{2}}.2$	$2^4.3^2$
*	$(t-1)^3$	1							2		
	$t-1$	1							2		
	$t+1$	2	0			1	δ		2^2	$3^{1+\frac{1}{2}}.2$	$2^4.3^2$
†*	$(t-1)^3$	1							2		
	$t-1$	1							2		
	$t+1$	2	ω			δ	δ		2^3	$3^{1+\frac{1}{2}}.2$	$2^4.3^2$
†*	$(t-1)^3$	1							2		
	$t-1$	1							2		
	$t+1$	2	ω			1	1		2^3	$3^{1+\frac{1}{2}}.2$	$2^4.3^2$
†*	$t+1$	4							$2^7.3^2$		
	$t-1$	2	0			0			2^2		$2^9.3^2$

TABLE 6 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	ψ_1^+	ψ_3^+	ψ_5^+	ψ_1^-	ψ_3^-	ψ_5^-	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
*	t+1	4							$2^5.3^2.5$		
	t-1	2	ω			ω			2^3		$2^8.3^2.5$
†*	$(t+1)^2$	2							$3^{-1}2^3.3$		
	t-1	2				0			2^2	$3^{\frac{1}{2}}.2^2$	$2^5.3^2$
†*	$(t+1)^3$	1							2		
	t+1	1							2		
	t-1	2	δ	1		0			2^2	$3^{1+\frac{1}{2}}.2$	$2^4.3^2$
†*	$(t+1)^3$	1							2		
	t+1	1							2		
	t-1	2	1	δ		0			2^2	$3^{1+\frac{1}{2}}.2$	$2^4.3^2$
*	$(t+1)^3$	1							2		
	t+1	1							2		
	t-1	2	δ	δ		ω			2^3	$3^{1+\frac{1}{2}}.2$	$2^5.3^2$
*	$(t+1)^3$	1							2		
	t+1	1							2		
	t-1	2	1	1		ω			2^3	$3^{1+\frac{1}{2}}.2$	$2^5.3^2$
	t-1	3							$2^4.3$		
	t+1	3	δ			1			$2^4.3$		$2^8.3^3$
†	t-1	3							$2^4.3$		
	t+1	3	1			δ			$2^4.3$		$2^8.3^2$
	$(t-1)^3$	1							2		
	t+1	3	δ				1		$2^4.3$	3	$2^5.3^2$
†	$(t-1)^3$	1							2		
	t+1	3	1				δ		$2^4.3$	3	$2^5.3^2$

TABLE 6 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	ψ_1^+	ψ_3^+	ψ_5^+	ψ_1^-	ψ_3^-	ψ_5^-	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
†	$(t+1)^3$	1							2		
	$t-1$	3		1		δ			$2^4.3$	3	$2^5.3^2$
	$(t+1)^3$	1							2		
	$t-1$	3		δ		1			$2^4.3$	3	$2^5.3^2$
†	$(t-1)^3$	1							2		
	$(t+1)^3$	1		1			δ		2	3.3	$2^2.3^2$
	$(t-1)^3$	1							2		
	$(t+1)^3$	1		δ			1		2	3.3	$2^2.3^2$
*	$t-1$	4							$2^5.3^2.5$		
	t^2+1	1				ω			2^2		$2^7.3^2.5$
*	$(t-1)^3$	1							2		
	$t-1$	1							2		
	t^2+1	1				δ	δ		2^2	$3^{1+\frac{1}{2}.2}$	$2^4.3^2$
*	$(t-1)^3$	1							2		
	$t-1$	1							2		
	t^2+1	1				1	1		2^2	$3^{1+\frac{1}{2}.2}$	$2^4.3^2$
†*	$t+1$	4							$2^5.3^2.5$		
	t^2+1	1	ω						2^2		$2^7.3^2.5$
†*	$(t+1)^3$	1							2		
	$t+1$	1							2		
	t^2+1	1	δ	δ					2^2	$3^{1+\frac{1}{2}.2}$	$2^4.3^2$
†*	$(t+1)^3$	1							2		
	$t+1$	1							2		
	t^2+1	1	1	1					2^2	$3^{1+\frac{1}{2}.2}$	$2^4.3^2$

TABLE 6 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	ψ_1^+	ψ_3^+	ψ_5^+	ψ_1^-	ψ_3^-	ψ_5^-	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
	$t-1$	3							$2^4.3$		
	$t+1$	1							2		
	t^2+1	1	1			1			2^2		$2^7.3$
†	$t-1$	3							$2^4.3$		
	$t+1$	1							2		
	t^2+1	1	δ			δ			2^2		$2^7.3$
	$(t-1)^3$	1							2		
	$t+1$	1							2		
	t^2+1	1	1				1		2^2	3	$2^4.3$
†	$(t-1)^3$	1							2		
	$t+1$	1							2		
	t^2+1	1	δ				δ		2^2	3	$2^4.3$
	$t+1$	3							$2^4.3$		
	$t-1$	1							2		
	t^2+1	1	δ			δ			2^2		$2^7.3$
†	$t+1$	3							$2^4.3$		
	$t-1$	1							2		
	t^2+1	1	1			1			2^2		$2^7.3$
	$(t+1)^3$	1							2		
	$t-1$	1							2		
	t^2+1	1		1		1			2^2	3	$2^4.3$
†	$(t+1)^3$	1							2		
	$t-1$	1							2		
	t^2+1	1		δ		δ			2^2	3	$2^4.3$

TABLE 6 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+ \ \psi_3^+ \ \psi_5^+$	$\psi_1^- \ \psi_3^- \ \psi_5^-$	$\Lambda(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
†*	$t-1$ $t+1$ t^2+1	2 2 1	0	ω	2^3 2^2 2^2		2^7
*	$t-1$ $t+1$ t^2+1	2 2 1	ω	0	2^2 2^3 2^2		2^7
†*	$t-1$ t^2+1	2 2		0	2^2 $2^5.3$		$2^7.3$
*	$t+1$ t^2+1	2 2	0		2^2 $2^5.3$		$2^7.3$
†	$t-1$ $t+1$ t^2+1	1 1 2	δ	1	2 2 $2^5.3$		$2^7.3$
	$t-1$ $t+1$ t^2+1	1 1 2	1	δ	2 2 $2^5.3$		$2^7.3$
†*	$t-1$ $(t^2+1)^2$	2 1		0	2^2 2^2	$9^{\frac{1}{2}}$	$2^4.3$
*	$t+1$ $(t^2+1)^2$	2 1	0		2^2 2^2	$9^{\frac{1}{2}}$	$2^4.3$
†	$t-1$ $t+1$ $(t^2+1)^2$	1 1 1	δ	1	2 2 2^2	$9^{\frac{1}{2}}$	$2^4.3$
	$t-1$ $t+1$ $(t^2+1)^2$	1 1 1	1	δ	2 2 2^2	$9^{\frac{1}{2}}$	$2^4.3$

TABLE 6 (Contd.)

	el.div. of X φ^{μ} $m(\varphi^{\mu})$	ψ_1^+ ψ_3^+ ψ_5^+	ψ_1^- ψ_3^- ψ_5^-	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
*	$t-1$ 2 $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$ 1		0	2^2 2^3		2^5
†*	$t+1$ 2 $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$ 1	0		2^2 2^3		2^5
	$t+1$ 1 $t-1$ 1 $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$ 1	δ	1	2 2 2^3		2^5
†	$t+1$ 1 $t-1$ 1 $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$ 1	1	δ	2 2 2^3		2^5
†*	$t-1$ 2 $t^4+t^3+t^2$ $+t+1$ 1		ω	2^3 2.5		$2^4.5$
†*	$t+1$ 2 $t^4+t^3+t^3$ $+t+1$ 1	ω		2^3 2.5		$2^4.5$
†	$t+1$ 1 $t-1$ 1 $t^4+t^3+t^2$ $+t+1$ 1	δ	δ	2 2 2.5		$2^3.5$
	$t+1$ 1 $t-1$ 1 $t^4+t^3+t^2$ $+t+1$ 1	1	1	2 2 2.5		$2^3.5$

TABLE 6 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+ \ \psi_3^+ \ \psi_5^+$	$\psi_1^- \ \psi_3^- \ \psi_5^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O^+(6,3)}(X)\rvert$
*	$t-1$ $t^4-t^3+t^2-t+1$	2 1		ω	2^3 2.5		$2^4.5$
*	$t+1$ $t^4-t^3+t^2-t+1$	2 1	ω		2^3 2.5		$2^4.5$
	$t+1$ $t+1$ $t-1$ $t^4-t^3+t^2-t+1$	1 1 1 1	δ	δ	2 2 2.5		$2^3.5$
†	$t+1$ $t-1$ $t^4-t^3+t^2-t+1$	1 1 1	1	1	2 2 2.5		$2^3.5$
*	t^2+1 $t^4+t^3+t^2+t+1$	1 1			2^2 2.5		$2^3.5$
†*	t^2+1 $t^4-t^3+t^2-t+1$	1 1			2^2 2.5		$2^3.5$
*α	$\left.\begin{matrix} t^3-t+1 \\ t^3-t^2+1 \end{matrix}\right\}$	1			2.13		2.13
†*α	$\left.\begin{matrix} t^3-t-1 \\ t^3+t^2-1 \end{matrix}\right\}$	1			2.13		2.13
*α	$\left.\begin{matrix} t^3+t^2-t+1 \\ t^3-t^2+t+1 \end{matrix}\right\}$	1			2.13		2.13
†*α	$\left.\begin{matrix} t^3+t^2+t-1 \\ t^3-t^2-t-1 \end{matrix}\right\}$	1			2.13		2.13

The conjugacy classes of $O^+(6,3)$ which lie in $SO\,(6,3)$ are marked by *.

The conjugacy classes which lie in $\Omega^+(6,3)$ are marked by †.

The non-trivial central element lies in $O^+(6,3)\backslash\Omega^+(6,3)$. Therefore no conjugacy classes of $O^+(6,3)$ split in $\Omega^+(6,3)$.

The conjugacy classes whose centralizer consists entirely of matrices of determinant 1 are marked by α. Hence those classes of $O^+(6,3)$ which are marked by α and which lie in $S\Omega\,(6,3) = PS\Omega^+(6,3)$ split into 2 conjugacy classes in $S\Omega^+(6,3)$.

Hence $PS\Omega^+(6,3)$ has exactly 29 conjugacy classes.

$PS\Omega^+(6,3) \cong PSL(4,3)$.

$|GL(4,3)| = (3^4-1)(3^4-3)(3^4-3^2)(3^4-3^3) = 2^9.3^6.5.13$

$|SL(4,3)| = 2^8.3^6.5.13$

$|PSL(4,3)| = 2^7.3^6.5.13$.

TABLE 7 Conjugacy classes in PSL(4,3)

el.div. of X φ^μ	 $m(\varphi^\mu)$	$\lvert C_{PSL(4,3)}(X)\rvert$	$o(X)$
$t-1$	4	$2^7.3^6.5.13$	1
$t-1$ $(t-1)^2$	2 1	$2^3.3^6$	3
$(t-1)^2$	2	$2^3.3^5$	3
$(t-1)^2$	2	$2^3.3^5$	3
$(t-1)^3$ $t-1$	1 1	3^4	3

TABLE 7 (Contd.)

el.div. of X φ^{μ} $m(\varphi^{\mu})$	$\lvert C_{PSL(4,3)}(X)\rvert$	o(X)
$(t-1)^4$ 1	3^3	9
$(t-1)^4$ 1	3^3	9
$t-1$ 2 $t+1$ 2	$2^7.3^2$	2
$(t-1)^2$ 1 $t+1$ 2	$2^3.3^2$	6
$(t-1)^2$ 1 $(t+1)^2$ 1	$2^2.3^2$	6
$(t-1)^2$ 1 $(t+1)^2$ 1	$2^2.3^2$	6
$t-1$ 2 t^2+1 1	$2^5.3$	4
$(t-1)^2$ 1 t^2+1 1	$2^2.3$	12
$t+1$ 1 $t-1$ 1 t^2+t-1 1	2^3	8
t^2+t-1 1 t^2-t-1 1	2^5	4
$t-1$ 1 t^3-t-1 1	13	13
$t-1$ 1 t^3+t^2-1 1	13	13

TABLE 7 (Contd.)

el.div. of X φ^{μ}	 $m(\varphi^{\mu})$	$\lvert C_{PSL(4,3)}(X)\rvert$	$o(X)$
$t-1$ t^3+t^2+t-1	1 1	13	13
$t-1$ t^3-t^2-t-1	1 1	13	13
t^2+1	2	$2^6.3^2.5$	2
$(t^2+1)^2$	1	$2^3.3^2$	6
$(t^2+1)^2$	1	$2^3.3^2$	6
t^2+t-1	2	$2^5.3^2.5$	4
$(t^2+t-1)^2$	1	$2^2.3^2$	12
$(t^2+t-1)^2$	1	$2^2.3^2$	12
t^4+t^2+t+1	1	$2^2.5$	20
t^4+t^3-t+1	1	$2^2.5$	10
$t^4+t^3+t^2+1$	1	$2^2.5$	20
t^4+t^3+ t^2+t+1	1	$2^2.5$	5

The map between conjugacy classes in PSL(4,3) and PSΩ(6,3) is uniquely determined up to conjugate pairs of conjugacy classes.

§13 Characters of $PS\Omega^+(6,3)$

$PSL(4,3)$ acts on $PG(4,3)$.

There are $1+3+3^2+3^3 = 40$ points in $PG(4,3)$. This is a doubly transitive representation of degree 40. Hence it yields a character of degree $39 = \beta_2$ s

There are $40.39/4.3 = 130$ lines in $PG(4,3)$. The group is a rank 3 group on these 130 lines and the corresponding character splits as $1 + \beta_2 + \beta_3$, β_3 a character of degree 90.

$|PS\Omega^+(6,3) : PS\Omega(5,3)| = 234$. Indeed there are 2 conjugacy classes of subgroups isomorphic to $PS\Omega(5,3)$ in $PS\Omega^+(6,3)$. The group is a rank 5 permutation group on the set of 234 conjugates and the character splits as

$$1 + \beta_3 + \beta_4 + \beta_5 + \beta_6.$$

The degree β_4 + degree β_5 + degree $\beta_6 = 143$.

By restricting and splitting this character on $PSL(3,3)$ and $PS\Omega(5,3)$ the degrees of the 3 characters are found to be 26, 52 and 65. The values of the characters on all conjugacy classes can also easily be found.

The remaining 21 characters can easily be found by inducing characters from $PS\Omega(5,3)$ and by forming tensor products from the characters of small degree already calculated. The complete character table is presented as TABLE 8.

TABLE 8

CHARACTER TABLE OF $PSL(4,3) \cong P S\Omega^{+}(6,3)$ OF ORDER 6,065,280.

$o(x)$	1	2	2	4	4	4	8	3	3	3	3	9	9	6	6	6	6	6	12	12	12	5	10	20	20	13	13	13	13
$\lvert C_{PSL(4,3)}(x)\rvert$	$2^7.3^6.5.13$ 6,065,280	$2^7.3^2$ 1152	$2^6.3^2.5$ 2880	$2^5.3$ 96	$2^5.3^2.5$ 1440	2^5 32	2^3 8	$2^3.3^6$ 5832	$2^3.3^5$ 1944	$2^3.3^5$ 1944	3^4 81	3^3 27	3^3 27	$2^3.3^2$ 72	$2^2.3^2$ 36	$2^2.3^2$ 36	$2^3.3^2$ 72	$2^3.3^2$ 72	$2^2.3$ 12	$2^2.3^2$ 36	$2^2.3^2$ 36	$2^2.5$ 20	$2^2.5$ 20	$2^2.5$ 20	$2^2.5$ 20	13 13	13 13	13 13	13 13
$\lvert cl(x)\rvert$	1 1	$3^4.5.13$ 5265	$2.3^4.13$ 2106	$2^2.3^5.5.13$ 63180	$2^2.3^4.13$ 4212	$2^2.3^6.5.13$ 189540	$2^4.3^6.5.13$ 758160	$2^4.5.13$ 1040	$2^4.3.5.13$ 3120	$2^4.3.5.13$ 3120	$2^7.3^2.5.13$ 74880	$2^7.3^3.5.13$ 224640	$2^7.3^3.5.13$ 224640	$2^4.3^4.5.13$ 84240	$2^5.3^4.5.13$ 168480	$2^5.3^4.5.13$ 168480	$2^4.3^4.5.13$ 84240	$2^4.3^4.5.13$ 84240	$2^5.3^5.5.13$ 505440	$2^5.3^4.5.13$ 168480	$2^5.3^4.5.13$ 168480	$2^5.3^6.13$ 303264	$2^5.3^6.13$ 303264	$2^5.3^6.13$ 303264	$2^5.3^6.13$ 303264	$2^7.3^6.5$ 466560	$2^7.3^6.5$ 466560	$2^7.3^6.5$ 466560	$2^7.3^6.5$ 466560
rep x	$t-1$ 4	$t-1$ 2, $t+1$ 2	t^2+1 2	$t-1$ 2, t^2+1 1	t^2+t-1 2	t^2+t-1, t^2-t-1	$t-1$ 1, $t+1$ 1, t^2+t-1 1	$t-1$ 2, $(t-1)^2$ 1	$(t-1)^2$ 2	$(t-1)^2$ 2	$(t-1)^3$ 1, $t-1$ 1	$(t-1)^4$ 1	$(t-1)^4$ 1	$(t-1)^2$ 1, $t+1$ 2	$(t-1)^2$ 1, $(t+1)^2$ 1	$(t-1)^2$ 1, $(t+1)^2$ 1	$(t^2+1)^2$	$(t^2+1)^2$	$(t-1)^2$ 1, t^2+1 1	$(t^2+t-1)^2$	$(t^2+t-1)^2$	$t^4+t^3+t^2+t+1$	$t^4-t^3+t^2-t+1$	$t^4+t^3+t^2+1$	t^4+t^3+t+1	$t-1$, t^3-t-1	$t-1$, t^3+t^2-1	$t-1$, t^3+t^2+t-1	$t-1$, t^3-t^2 [illegible]
β_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
β_2	26	2	6	2	4	0	0	-1	-1	8	-1	2	-1	-1	-1	2	3	0	-1	-2	1	1	1	-1	-1	0	0	0	0
β_3	26	2	6	2	4	0	0	-1	8	-1	-1	-1	2	-1	2	-1	0	3	-1	1	-2	1	1	-1	-1	0	0	0	0
β_4	39	7	-1	3	-1	-1	1	12	3	3	3	0	0	4	1	1	-1	-1	0	-1	-1	-1	-1	-1	-1	0	0	0	0
β_5	52	-4	8	0	-10	2	0	-2	7	7	-2	1	1	2	-1	-1	-1	-1	0	-1	-1	2	-2	0	0	0	0	0	0
β_6	65	-7	5	1	-5	-1	-1	11	2	11	2	2	-1	-1	2	-1	2	-1	1	1	-2	0	0	0	0	0	0	0	0
β_7	65	-7	5	1	-5	-1	-1	11	11	2	2	-1	2	-1	-1	2	-1	2	1	-2	1	0	0	0	0	0	0	0	0
β_8	90	10	10	-2	10	2	0	9	9	9	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	-1	-1	-1	-1
β_9	234	2	14	2	-4	0	0	-9	18	-9	0	0	0	-1	2	-1	2	-1	-1	-1	2	-1	-1	1	1	0	0	0	0
β_{10}	234	2	14	2	-4	0	0	-9	-9	18	0	0	0	-1	-1	2	-1	2	-1	2	-1	-1	-1	1	1	0	0	0	0
β_{11}	260	-4	0	0	10	-2	0	-10	17	-1	-1	2	-1	2	-1	-1	-3	3	0	1	1	0	0	0	0	0	0	0	0
β_{12}	260	-4	0	0	10	-2	0	-10	-1	17	-1	-1	2	2	-1	-1	3	-3	0	1	1	0	0	0	0	0	0	0	0
β_{13}	260	4	20	4	0	0	0	17	-10	-10	-1	-1	-1	1	-2	-2	2	2	1	0	0	0	0	0	0	0	0	0	0
β_{14}	351	15	-9	-1	-9	-1	-1	27	0	0	0	0	0	3	0	0	0	0	-1	0	0	1	1	1	1	0	0	0	0
β_{15}	390	-10	-10	2	10	2	0	39	3	3	3	0	0	-1	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0	0	0
β_{16}	416	0	16	0	16	0	0	-16	2	2	2	-1	-1	0	0	0	-2	-2	0	-2	-2	1	1	1	1	0	0	0	0
β_{17}	416	0	16	0	-16	0	0	-16	2	2	2	-1	-1	0	0	0	-2	-2	0	2	2	1	1	-1	-1	0	0	0	0
β_{18}	416	0	-16	0	0	0	0	-16	2	2	2	-1	-1	0	0	0	2	2	0	0	0	1	-1	$i\sqrt{5}$	$-i\sqrt{5}$	0	0	0	0
β_{19}	416	0	-16	0	0	0	0	-16	2	2	2	-1	-1	0	0	0	2	2	0	0	0	1	1	$-i\sqrt{5}$	$i\sqrt{5}$	0	0	0	0
β_{20}	468	-4	-8	0	-10	2	0	-18	9	9	0	0	0	2	-1	-1	1	1	0	-1	-1	-2	2	0	0	0	0	0	0
β_{21}	585	1	5	-3	-5	-1	1	18	-9	18	0	0	0	-2	1	-2	-1	2	0	-2	1	0	0	0	0	0	0	0	0
β_{22}	585	1	5	-3	-5	-1	1	18	18	-9	0	0	0	-2	-2	1	2	-1	0	1	-2	0	0	0	0	0	0	0	0
β_{23}	640	0	0	0	0	0	0	-8	-8	-8	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	$\varepsilon+\varepsilon^3+\varepsilon^9$	$\varepsilon^{12}+\varepsilon^{10}+\varepsilon^4$	$\varepsilon^2+\varepsilon^5+\varepsilon^6$	$\varepsilon^{11}+\varepsilon^8+\varepsilon^7$
β_{24}	640	0	0	0	0	0	0	-8	-8	-8	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	$\varepsilon^{12}+\varepsilon^{10}+\varepsilon^4$	$\varepsilon+\varepsilon^3+\varepsilon^9$	$\varepsilon^{11}+\varepsilon^8+\varepsilon^7$	$\varepsilon^2+\varepsilon^5+\varepsilon^6$
β_{25}	640	0	0	0	0	0	0	-8	-8	-8	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	$\varepsilon^2+\varepsilon^5+\varepsilon^6$	$\varepsilon^{11}+\varepsilon^8+\varepsilon^7$	$\varepsilon+\varepsilon^3+\varepsilon^9$	$\varepsilon^{12}+\varepsilon^{10}+\varepsilon^4$
β_{26}	640	0	0	0	0	0	0	-8	-8	-8	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	$\varepsilon^{11}+\varepsilon^8+\varepsilon^7$	$\varepsilon^2+\varepsilon^5+\varepsilon^6$	$\varepsilon^{12}+\varepsilon^{10}+\varepsilon^4$	$\varepsilon+\varepsilon^3+\varepsilon^9$
β_{27}	729	9	9	-3	9	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	1	1
β_{28}	780	12	-20	4	0	0	0	-3	6	6	-3	0	0	-3	0	0	-2	-2	1	0	0	0	0	0	0	0	0	0	0
β_{29}	1040	-16	0	0	0	0	0	14	-4	-4	-4	-1	-1	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0

§14 Conjugacy classes in $PS\Omega(7,3)$

We apply Theorem 11.2 to find the conjugacy classes in $O(7,3)$. We list below the conjugacy classes of $O(7,3)$ contained in $SO(7,3)$. We note that the element

$$\begin{bmatrix} -1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & -1 & & & \\ & & & & -1 & & \\ & & & & & -1 & \end{bmatrix}$$ is central and has determinant -1.

Therefore $2|C_{SO(7,3)}(X)| = |C_{O(7,3)}(X)|$ for all $X \in O(7,3)$ and therefore no conjugacy classes split in $SO(7,3)$.

The conjugacy classes of $SO(7,3)$ which lie in $\Omega(7,3)$ are marked by †.

The conjugacy classes for which $|C_{SO(7,3)}(X)| = |C_{S\Omega(7,3)}(X)|$ are marked by *, i.e. these classes yield 2 conjugacy classes in $S\Omega(7,3)$.

For some classes it is immediately clear that $|C_{SO(7,3)}(X)| = 2\,|C_{S\Omega(7,3)}(X)|$. e.g. those for which the elementary divisor $t+1$ or $t-1$ occurs with multiplicity greater than or equal to 2.

For other classes it is necessary to construct actual elements with given elementary divisors. G.B. Elkington has shown me a way of achieving this aim by varying the orthogonal form to suit the element X.

e.g. Let X have the following elementary divisors and Witt invariants :=

$t+1$	1	δ
$(t+1)^5$	1	δ
$t-1$	1	δ

TABLE 9 Conjugacy classes in O(7,3)

	el.div. of X φ^{μ}	 $m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
†	$t-1$	7		1	$2^{11}.3^9.5.7.13$		$2^{11}.3^9.5.7.13$
†	$t-1$ $(t-1)^2$	3 2		 1	$2^4.3$ $3^{-1}2^3.3$	$3^{6+\frac{1}{2}}.2^2$	$2^7.3^9$
†	$t-1$ $(t-1)^3$	4 1		 0 1	$2^7.3^2$ 2	$3^{4+\frac{1}{2}}.2$	$2^8.3^7$
†	$t-1$ $(t-1)^3$	4 1		 ω δ	$2^5.3^2.5$ 2	$3^{4+\frac{1}{2}}.2$	$2^6.3^7.5$
*†	$(t-1)^2$ $(t-1)^3$	2 1		 1	$3^{-1}2^3.3$ 2	$3^{4+\frac{1}{2}2^2+1}$	$2^4.3^7$
†	$t-1$ $(t-1)^3$	1 2		 1 0	2 2^2	$3^{2+\frac{1}{2}2}.2^2$	$2^3.3^6$
†	$t-1$ $(t-1)^3$	1 2		 δ ω	2 2^3	$3^{2+\frac{1}{2}2}.2^2$	$2^4.3^6$
†	$t-1$ $(t-1)^5$	2 1		 0 1	2^2 2	$3^{2+\frac{1}{2}}.4$	$2^3.3^4$
†	$t-1$ $(t-1)^5$	2 1		 ω δ	2^3 2	$3^{2+\frac{1}{2}}.4$	$2^4.3^4$
*†	$(t-1)^7$	1		1	2	$3^{\frac{1}{2}}.6$	2.3^3
	$t+1$ $t-1$	6 1	 0	 1	$2^9.3^6.5.13$ 2		$2^{10}.3^6.5.13$
†	$t+1$ $t-1$	6 1	 ω	 δ	$2^9.3^6.5.7$ 2		$2^{10}.3^6.5.7$

TABLE 9 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
*†	$(t+1)^5$	1			2		
	$t+1$	1			2		
	$t-1$	1	δ δ	δ	2	$3^{1+\frac{1}{2}}.4$	$2^3.3^3$
†	$(t+1)^5$	1			2		
	$t+1$	1			2		
	$t-1$	1	1 1	δ	2	$3^{1+\frac{1}{2}}.4$	$2^3.3^3$
	$t-1$	5			$2^8.3^4.5$		
	$t+1$	2	0	1	2^2		$2^{10}.3^4.5$
†	$t-1$	5			$2^8.3^4.5$		
	$t+1$	2	ω	δ	2^3		$2^{11}.3^4.5$
	$t-1$	1			2		
	$(t-1)^2$	2			$3^{-1}2^3.3$		
	$t+1$	2	0	1	2^2	$3^{2+\frac{1}{2}}.2^2$	$2^6.3^4$
†	$t-1$	1			2		
	$(t-1)^2$	2			$3^{-1}2^3.3$		
	$t+1$	2	ω	δ	2^3	$3^{2+\frac{1}{2}}.2^2$	$2^7.3^4$
	$(t-1)^3$	1			2		
	$t-1$	2			2^2		
	$t+1$	2	0	0 1	2^2	$3^{2+\frac{1}{2}}.2$	$2^5.3^3$
†	$(t-1)^3$	1			2		
	$t-1$	2			2^3		
	$t+1$	2	ω	ω 1	2^3	$3^{2+\frac{1}{2}}.2$	$2^7.3^3$
†	$(t-1)^3$	1			2		
	$t-1$	2			2^3		
	$t+1$	1	ω	0 δ	2^2	$3^{2+\frac{1}{2}}.2$	$2^6.3^3$

TABLE 9 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$t+1$	2			2^2		
	$(t+1)^2$	2			$3^{-1}2^3.3$		
	$t-1$	1	0	1	2	$3^{4+\frac{1}{2}}.2^2$	$2^6.3^6$
†	$t+1$	2			2^3		
	$(t+1)^2$	2			$3^{-1}2^3.3$		
	$t-1$	1	ω	δ	2	$3^{4+\frac{1}{2}}.2^2$	$2^6.3^6$
	$(t+1)^3$	1			2		
	$t+1$	3			$2^4.3$		
	$t-1$	1	δ 1	1	2	$3^{3+\frac{1}{2}}.2$	$2^6.3^5$
	$(t+1)^3$	1			2		
	$t+1$	3			$2^4.3$		
	$t-1$	1	1 δ	1	2	$3^{3+\frac{1}{2}}.2$	$2^6.3^5$
†	$(t+1)^3$	1			2		
	$t+1$	3			$2^4.3$		
	$t-1$	1	δ δ	δ	2	$3^{3+\frac{1}{2}}.2$	$2^6.3^5$
†	$(t+1)^3$	1			2		
	$t+1$	3			$2^4.3$		
	$t-1$	1	1 1	δ	2	$3^{3+\frac{1}{2}}.2$	$2^6.3^5$
	$(t+1)^3$	2			2^2		
	$t-1$	1	0	1	2	$3^{\frac{1}{2}}.2.2^2$	$2^3.3^4$
†	$(t+1)^3$	2			2^3		
	$t-1$	1	ω	δ	2	3^4	$2^4.3^4$
	$(t+1)^5$	1			2		
	$t+1$	1			2		
	$t-1$	1	δ 1	1	2	$3^{1+\frac{1}{2}}.4$	$2^3.3^3$
	$(t+1)^5$	1			2		
	$t+1$	1			2		
	$t-1$	1	1 δ	1	2	$3^{1+\frac{1}{2}}.4$	$2^3.3^3$

TABLE 9 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$(t-1)^3$	1			2		
	$t-1$	2			2^2		
	$t+1$	2	0	ω δ	2^3	$3^{2+\frac{1}{2}}.2$	$2^6.3^3$
	$(t-1)^5$	1			2		
	$t+1$	2	0	1	2^2	$3^{\frac{1}{2}}.4$	$2^3.3^2$
†	$(t-1)^5$	1			2		
	$t+1$	2	ω	δ	2^3	$3^{\frac{1}{2}}.4$	$2^4.3^2$
†	$t+1$	4			$2^7.3^2$		
	$t-1$	3	0	1	$2^4.3$		$2^{11}.3^3$
	$t+1$	4			$2^5.3^2.5$		
	$t-1$	3	ω	δ	$2^4.3$		$2^9.3^3.5$
†	$(t+1)^2$	2			$3^{-1}2^3.3$		
	$t-1$	3		1	$2^4.3$	$3^{\frac{1}{2}}.2^2$	$2^7.3^3$
†	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	3	δ 1	1	$2^4.3$	$3^{1+\frac{1}{2}}.2$	$2^6.3^3$
	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	3	δ δ	δ	$2^4.3$	$3^{1+\frac{1}{2}}.2$	$2^6.3^3$
	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	3	1 1	δ	$2^4.3$	$3^{1+\frac{1}{2}}.2$	$2^6.3^3$
†	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	3	1 δ	1	$2^4.3$	$3^{1+\frac{1}{2}}.2$	$2^6.3^3$

TABLE 9 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$t+1$	4			$2^7.3^2$		
	$(t-1)^3$	1	0	1	2	$3^{\frac{1}{2}}.2$	$2^8.3^3$
†	$t+1$	4			$2^5.3^2.5$		
	$(t-1)^3$	1	ψ	δ	2	$3^{\frac{1}{2}}.2$	$2^6.3^3.5$
*†	$(t+1)^2$	2			$3^{-1}2^3.3$	$3^{\frac{1}{2}}.2^2$	
	$(t-1)^3$	1		1	2	$3^{\frac{1}{2}}.2$	$2^4.3^3$
†	$(t+1)^3$	1			2		
	$t+1$	1			2	$3^{1+\frac{1}{2}}.2$	
	$(t-1)^3$	1	δ 1	1	2	$3^{\frac{1}{2}}.2$	$2^3.3^3$
	$(t+1)^3$	1			2		
	$t+1$	1			2	$3^{1+\frac{1}{2}}.2$	
	$(t-1)^3$	1	δ δ	δ	2	$3^{\frac{1}{2}}.2$	$2^3.3^3$
	$(t+1)^3$	1			2		
	$t+1$	1			2	$3^{1+\frac{1}{2}}.2$	
	$(t-1)^3$	1	1 1	δ	2	$3^{\frac{1}{2}}.2$	$2^3.3^3$
*†	$(t+1)^3$	1			2		
	$t+1$	1			2	$3^{1+\frac{1}{2}}.2$	
	$(t-1)^3$	1	1 δ	1	2	$3^{\frac{1}{2}}.2$	$2^3.3^3$
	$t-1$	5			$2^8.3^4.5$		
	t^2+1	1		δ	2^2		$2^{10}.3^4.5$
	$(t-1)^2$	2			$3^{-1}2^3.3$		
	$t-1$	1			2		
	t^2+1	1		δ	2^2	$3^{2+\frac{1}{2}}.2^2$	$2^6.3^4$
	$t-1$	2			2^2		
	$(t-1)^3$	1			2		
	t^2+1	1		0 δ	2^2	$3^{2+\frac{1}{2}}.2$	$2^5.3^3$

TABLE 9 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$t-1$	2			2^3		
	$(t-1)^3$	1			2		
	t^2+1	1		ω 1	2^2	$3^{2+\frac{1}{2}}\cdot 2$	$2^6\cdot 3^3$
	$(t-1)^5$	1			2		
	t^2+1	1		δ	2^2	$3^{\frac{1}{2}}\cdot 4$	$2^3\cdot 3^2$
	$t+1$	4			$2^7\cdot 3^2$		
	$t-1$	1			2		
	t^2+1	1	0	δ	2^2		$2^{10}\cdot 3^2$
†	$t+1$	4			$2^5\cdot 3^2\cdot 5$		
	$t-1$	1			2		
	t^2+1	1	ω	1	2^2		$2^8\cdot 3^2\cdot 5$
	$(t+1)^2$	2			$3^{-1}2^3\cdot 3$		
	$t-1$	1			2		
	t^2+1	1		δ	2^2	$3^{\frac{1}{2}}\cdot 2^2$	$2^6\cdot 3^2$
†	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	1			2		
	t^2+1	1	1 1	1	2^2	$3^{1+\frac{1}{2}}\cdot 2$	$2^5\cdot 3^2$
†	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	1			2		
	t^2+1	1	δ δ	1	2^2	$3^{1+\frac{1}{2}}\cdot 2$	$2^5\cdot 3^2$
	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	1			2		
	t^2+1	1	1 δ	δ	2^2	$3^{1+\frac{1}{2}}\cdot 2$	$2^5\cdot 3^2$

TABLE 9 (Contd.)

	el.div. of X φ^μ	$m(\varphi^\mu)$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^\mu)$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$(t+1)^3$	1			2		
	$t+1$	1			2		
	$t-1$	1			2		
	t^2+1	1	δ 1	δ	2^2	$3^{1+\frac{1}{2}}.2$	$2^5.3^2$
†	$t-1$	3			$2^4.3$		
	$t+1$	2			2^2		
	t^2+1	1	0	δ	2^2		$2^8.3$
	$t-1$	3			$2^4.3$		
	$t+1$	2			2^3		
	t^2+1	1	ω	1	2^2		$2^9.3$
†	$(t-1)^3$	1			2		
	$t+1$	2			2^2		
	t^2+1	1	0	δ	2^2	$3^{\frac{1}{2}}.2$	$2^5.3$
	$(t-1)^3$	1			2		
	$t+1$	2			2^3		
	t^2+1	1	ω	1	2^2	$3^{\frac{1}{2}}.2$	$2^6.3$
†	$t-1$	3			$2^4.3$		
	t^2+1	2		1	$2^5.3$		$2^9.3^2$
†	$(t-1)^3$	1			2		
	t^2+1	2		1	$2^5.3$	$3^{\frac{1}{2}}.2$	$2^6.3^2$
	$t+1$	2			2^2		
	$t-1$	1			2		
	t^2+1	2	0	1	$2^5.3$		$2^8.3$
	$t+1$	2			2^3		
	$t-1$	1			2		
	t^2+1	2	ω	δ	$2^5.3$		$2^9.3$

<u>TABLE 9 (Contd.)</u>

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
†	$t-1$ $(t^2+1)^2$	3 1		1	$2^4.3$ 2^2	$9^{\frac{1}{2}}$	$2^6.3^2$
*†	$(t-1)^3$ $(t^2+1)^2$	1 1		1	2 2^2	$3^{\frac{1}{2}}.2$ $9^{\frac{1}{2}}$	$2^3.3^2$
	$t+1$ $t-1$ $(t^2+1)^2$	2 1 1	0	1	2^2 2 2^2	$9^{\frac{1}{2}}$	$2^5.3$
†	$t+1$ $t-1$ $(t^2+1)^2$	2 1 1	ω	δ	2^3 2 2^2	$9^{\frac{1}{2}}$	$2^6.3$
	$t-1$ t^2+1	1 3		δ	2 $2^7.3^2.7$		$2^8.3^3.7$
	$t-1$ $(t^2+1)^2$ t^2+1	1 1 1		δ	2 2^2 2^2	$9^{1+\frac{1}{2}}$	$2^5.3^3$
	$t-1$ $(t^2+1)^3$	1 1		δ	2 2^2	$9^{\frac{1}{2}}.2$	$2^3.3^2$
	$t-1$ $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$	3 1		1	$2^4.3$ 2^3		$2^7.3$
	$(t-1)^3$ $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$	1 1		1	2 2^3	$3^{\frac{1}{2}}.2$	$2^4.3$
†	$t+1$ $t-1$ $\left.\begin{matrix}t^2+t-1\\t^2-t-1\end{matrix}\right\}$	2 1 1	0	1	2^2 2 2^3		2^6

<u>TABLE 9 (Contd.)</u>

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$t+1$	2			2^3		
	$t-1$	1			2		
	$\left.\begin{matrix} t^2+t-1 \\ t^2-t-1 \end{matrix}\right\}$	1	ω	δ	2^3		2^7
†	$t-1$	3			$2^4.3$		
	$t^4+t^3+t^2+t+1$	1		δ	2.5		$2^5.3.5$
†	$(t-1)^3$	1			2		
	$t^4+t^3+t^2+t+1$	1		δ	2.5	$3^{\frac{1}{2}}.2$	$2^2.3.5$
	$t+1$	2			2^2		
	$t-1$	1			2		
	$t^4+t^3+t^2+t+1$	1	0	δ	2.5		$2^4.5$
†	$t+1$	2			2^3		
	$t-1$	1			2		
	$t^4+t^3+t^2+t+1$	1	ω	1	2.5		$2^5.5$
	$t-1$	3			$2^4.3$		
	$t^4-t^3+t^2-t+1$	1		δ	2.5		$2^5.3.5$
	$(t-1)^3$	1		δ	2		
	$t^4-t^3+t^2-t+1$	1		δ	2.5	$3^{\frac{1}{2}}.2$	$2^2.3.5$
†	$t+1$	2			2^2		
	$t-1$	1			2		
	$t^4-t^3+t^2-t+1$	1	0	δ	2.5		$2^4.5$

TABLE 9 (Contd.)

	el.div. of X φ^{μ}	$m(\varphi^{\mu})$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^{\mu})$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
	$t+1$ $t-1$ $t^4-t^3+t^2-t+1$	2 1 1	ω	1	2^3 2 2.5		$2^5.5$
†	$t-1$ t^2+1 $\left.\begin{matrix}t^2+t-1\\ t^2-t-1\end{matrix}\right\}$	1 1 1		δ	2 2^2 2^3		2^6
	$t-1$ t^2+1 $t^4+t^3+t^2+t+1$	1 1 1		1	2 2^2 2.5		$2^4.5$
†	$t-1$ t^2+1 $t^4-t^3+t^2-t+1$	1 1 1		1	2 2^2 2.5		$2^4.5$
	$t-1$ $\left.\begin{matrix}t^3-t^2+1\\ t^3-t+1\end{matrix}\right\}$	1 1		1	2 2.13		$2^2.13$
†	$t-1$ $\left.\begin{matrix}t^3-t-1\\ t^3+t^2-1\end{matrix}\right\}$	1 1			2 2.13		$2^2.13$
	$t-1$ $\left.\begin{matrix}t^3+t^2-t+1\\ t^3-t^2+t+1\end{matrix}\right\}$	1 1		1	2.13		$2^2.13$
†	$t-1$ $\left.\begin{matrix}t^3+t^2+t-1\\ t^3-t^2-t-1\end{matrix}\right\}$	1 1		1	2.13		$2^2.13$

TABLE 9 (Contd.)

el.div. of X φ^μ $\quad m(\varphi^\mu)$	$\psi_1^+\psi_3^+\psi_5^+$	$\psi_1^-\psi_3^-\psi_5^-\psi_7^-$	$A(\varphi^\mu)$	$B(\varphi)$	$\lvert C_{O(7,3)}(X)\rvert$
$t-1$ $\quad 1$ $t^6+t^5+t^3$ $+t+1$ $\quad 1$		δ	2 $2^2.7$		$2^3.7$
† $t-1$ $\quad 1$ $t^6+t^5+t^4$ $+t^3+t^2+$ $t+1$ $\quad 1$		δ	2 $2^2.7$		$2^3.7$
$t-1$ $\quad 1$ $t^6-t^5-t^3$ $-t+1$ $\quad 1$		δ	2 $2^2.7$		$2^3.7$
† $t-1$ $\quad 1$ t^6-t^5+ $t^4-t^3+t^2$ $-t+1$ $\quad 1$		δ	2 $2^2.7$		$2^3.7$

The matrix $\left[\begin{array}{c:cc:cc} -1 & & & & 1 \\ \hdashline 1 & -1 & -1 & & 1 \\ 1 & & -1 & & 1 \\ \hdashline & & & -1 & \\ & & & 1 & -1 \end{array}\right] = X_2$ with form $\left[\begin{array}{c:cc:cc} -1 & & & & \\ \hdashline & & & 1 & \\ & & & & 1 \\ \hdashline & 1 & & & \\ & & 1 & & \end{array}\right] = A_2$

has elementary divisor $(t+1)^5$.

The elements of order 2 in the centralizer of the element $X = \begin{bmatrix} -1 & \\ & X_{2_1} \end{bmatrix}$ with respect to the orthogonal form

$A \begin{bmatrix} -1 & & \\ & A_2 & \\ & & -1 \end{bmatrix}$ are $\begin{bmatrix} I_1 & & \\ & I_5 & \\ & & I_1 \end{bmatrix}$ $\begin{bmatrix} -I_1 & & \\ & I_5 & \\ & & -I_1 \end{bmatrix}$ $\begin{bmatrix} -I_1 & & \\ & -I_5 & \\ & & I_1 \end{bmatrix}$

and $\begin{bmatrix} I_1 & & \\ & -I_5 & \\ & & -I_1 \end{bmatrix}$. Now a useful rule is that $-I \in \Omega$ if and only if the determinant of the form is $+1$.

Therefore all 4 elements listed above ε $S\Omega(7,3)$ and hence $|C_{SO(7,3)}(X)| = |C_{S\Omega(7,3)}(X)|$. $\natural\natural$

Hence we have that $S\Omega(7,3) = PS\Omega(7,3)$ contains 58 conjugacy classes.

§15 Characters of $PS\Omega(7,3)$

We use subgroups isomorphic to $PS\Omega^+(6,3)$, $S\Omega^-(6,3)$, $PSp(6,2)$ and Σ_9 to calculate most of the characters of $PS\Omega(7,3)$ by inducing the characters of these subgroups. We also use the rank 3 representations on points of the projective plane $PG(7,3)$.

The number of points in $PG(7,3) = 1+3+3^2+3^3+3^4+3^5+3^6 = (3^7-1)/(3-1) = 1093$.

The number of length $0 = \binom{7}{3}.2^2 + 7.2^5 = 364$

The number of length $1 = 7 + \binom{7}{4}.2^3 + 2^6 = 351$

The number of length $-1 = \binom{7}{2}.2 + \binom{7}{5}.2^4 = 378$

$|PS\Omega(7,3) : S\Omega^-(6,3)| = 2.3^3.13 = 702$.

$S\Omega^-(6,3)$ is not a maximal subgroup of $PS\Omega(7,3)$ but $\Omega^-(6,3)$ is maximal of index 351. $\Omega^-(6,3)$ is the stabilizer of a point of length 1 in the projective plane.

Suppose $\Omega^-(6,3) = S\Omega^-(6,3) \cup M.S\Omega^-(6,3)$. Then if Y is an arbitrary matrix in $S\Omega^-(6,3)$ the group $\Omega^-(6,3)$ embeds in $PS\Omega(7,3)$ as follows :=

$$Y \to \left[\begin{array}{c|c} Y & \\ \hline & 1 \end{array}\right] \quad \text{with form} \quad \begin{bmatrix} & 1 & & & & & \\ 1 & & & & & & \\ & & & 1 & & & \\ & & 1 & & & & \\ & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & -1 \end{bmatrix}$$

$$MY \to \left[\begin{array}{c|c} MY & \\ \hline & -1 \end{array}\right] \quad \text{with form} \quad \begin{bmatrix} & 1 & & & & & \\ 1 & & & & & & \\ & & & 1 & & & \\ & & 1 & & & & \\ & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & -1 \end{bmatrix}$$

i.e. adjoin $t-1\ \underset{\sim}{\delta}$ or $t+1\ \underset{\sim}{\delta}$ according as the matrix has determinant 1 or -1.

The character table of $PS\Omega^{-}(6,3)$ has been presented as TABLE 4. The conjugacy classes of $\Omega^{-}(6,3)$ and $S\Omega^{-}(6,3)$ are calculated using Theorem 11.2 but as they are so similar to the earlier calculations for $O^{+}(6,3)$ and $O(7,3)$ these calculations are omitted.

Hence a character of degree 351, $(1_{\Omega^{-}(6,3)})^{PS\Omega(7,3)}$ and a character of degree 702, $(1_{S\Omega^{-}(6,3)})^{PS\Omega(7,3)}$ are calculated.

$|PS\Omega(7,3) : S\Omega^{+}(6,3)| = 2^2.3^3.7 = 756$. $S\Omega^{+}(6,3)$ is not a maximal subgroup of $PS\Omega(7,3)$ but $\Omega^{+}(6,3)$ is a maximal subgroup of index 378. This subgroup is embedded in a similar fashion to the embedding of $\Omega^{-}(6,3)$.

Hence a permutation character of degree 378 $(1_{\Omega^{+}(6,3)})^{PS\Omega(7,3)}$ and a character of degree 756 $(1_{S\Omega^{\ddagger}(6,3)})^{PS\Omega(7,3)}$ are calculated.

The representation of $PS\Omega(7,3)$ on the 1093 points of $PG(7,3)$ merely depends on the conjugacy class in $GL(7,3)$, i.e. on the elementary divisors and by subtraction the character of the representation on 364 isotropic points can be obtained. Inspection of the characters of the 3 rank 3 representations shows that :=

the character of degree $351 = \chi_1^{(7)} + \chi_a^{(7)} + \chi_b^{(7)}$.

the character of degree $364 = \chi_1^{(7)} + \chi_a^{(7)} + \chi_c^{(7)}$

the character of degree $378 = \chi_1^{(7)} + \chi_b^{(7)} + \chi_c^{(7)}$

Hence degree $\chi_a^{(7)} = 168$

degree $\chi_b^{(7)} = 182$

degree $\chi_c^{(7)} = 195$ and these 3 irreducible characters are determined on all conjugacy classes. (They are in fact $\chi_5^{(7)}, \chi_6^{(7)}$ and $\chi_7^{(7)}$. See TABLE 10.)

The representation on 702 symbols is rank 5 and the remaining 2 characters are $\chi_2^{(7)}$ and $\chi_{10}^{(7)}$ of degrees 78 and 273.

The representation on 756 symbols is rank 5 and the remaining 2 characters are $\chi_4^{(7)}$ and $\chi_{10}^{(7)}$ of degrees 105 and 273.

The values of these 3 characters can be obtained by restricting and splitting in $PS\Omega^+(6,3)$ and $S\Omega^-(6,3)$.

$PS\Omega^+(7,3)$ contains 2 conjugacy classes of subgroups isomorphic to $PSp(6,2) \cong W(E_7)$ of order $2^9.3^4.5.7$ and index $3^5.13 = 3159$. (Lemma 1.7). The character table of $Sp(6,2)$ is well-known and is published by J.S. Frame [4]. I have checked the table completely as there is one error, character 120_a and class number 11.

$PS\Omega(7,3)$ contains 351 d's. There are $351.126/2 = 3^3.13.3^2.7 = 3^5.7.13$ t's and $351.126.45/3.2.1 = 3^6.5.7.13$ n's. Therefore conjugacy classes no. 2,3 and 4 are D,T and N respectively. (See TABLE 10).

In $Sp(6,2)$ there are $3^2.7$ d's, $63.30/2.1 = 3^3.5.7$ t's and $63.30.13/3.2.1 = 3^2.5.7.13 = 3^2.5.7 + 2^2.3^3.5.7$ n's. We have therefore determined the D,T and N's in $Sp(6,2)$.

$\chi_2^{(7)}$ restricted to Sp(6,2) splits as $15_a+7_a+56_a$.

$\chi_3^{(7)}$ restricted to Sp(6,2) = 105_a. i.e. remains irreducible.

This solves the fusion problem except for conjugacy classes nos. 7,10,11,12,13,14,23 and 24.

Restricting $\chi_6^{(7)}$ determines the image of nos. 7 and 24. The remaining 6 conjugacy classes can be mapped to either of 2 conjugacy classes in PSΩ(7,3). This choice corresponds to the 2 conjugacy classes of subgroups isomorphic to Sp(6,2).

In an entirely analagous fashion it is possible to embed the 2 conjugacy classes of subgroups isomorphic to Σ_9, the symmetric group on 9 symbols. (Lemma 1.7.) The character table of Σ_9 is in Littlewood [13].

PSΩ(7,3) acts as a rank 4 group of degree 3159 on the conjugates of Sp(6,2) and a rank 5 group of degree $2^2.3^5.13 = 12636$ on the conjugates of Σ_9.

The remainder of the 58 characters of PSΩ(7,3) were calculated by inducing characters from the various subgroups discussed in this section and forming tensor products of characters of small degree. The complete character table is presented as TABLE 10. Unfortunately the numbering of the conjugacy classes used in later sections is not quite consecutive but runs from 1 to 36, 58 then 37 to 57.

TABLE 10

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
o(x)	1	2	2	2	4	4	4	4	8	8	3	3	3	3	3	3	3	9	9	9
|C(x)|	$2^9 3^9 5\cdot7\cdot13$	$2^9 3^6 5\cdot7$	$2^9 3^4 5$	$2^9 3^3$	$2^6 3^2 5$	$2^7 3^2$	$2^7 3$	$2^6 3$	2^4	2^4	$2^4 3^9$	$2^6 3^7$	$2^4 3^7 5$	$2^3 3^7$	$2^3 3^7$	$2\cdot3^6$	$2^2 3^6$	$2\cdot3^4$	$2^2 3^4$	3^3
|cl(x)|	1	351	22113	331,695	1,592,136	3,980,340	11,941,020	23,882,040	286,584,480	286,584,480	2280	32,760	26,208	262,080	262,080	3,144,960	1,572,480	28,304,640	14,152,320	169,827,840
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	78	-34	14	-2	-6	2	2	2	0	0	-3	6	15	-3	-3	-3	6	0	3	0
χ_3	91	-21	11	-5	-1	3	3	-1	-1	-1	10	19	1	10	10	1	1	4	-2	1
χ_4	105	-35	5	1	-5	5	1	-1	1	-1	24	6	15	-3	-3	6	-3	0	3	0
χ_5	168	56	24	8	4	0	0	4	0	0	6	24	15	6	6	-3	6	3	0	0
χ_6	182	70	22	6	10	2	2	2	0	0	20	11	29	-7	-7	2	2	-1	5	-1
χ_7	195	55	15	11	5	3	-1	1	1	-1	33	24	15	6	6	6	-3	3	0	0
χ_8	260	20	20	4	0	4	4	0	0	0	17	-10	-10	-10	17	-1	8	-1	-1	-1
χ_9	260	20	20	4	0	4	4	0	0	0	17	-10	-10	17	-10	-1	8	-1	-1	2
χ_{10}	273	-91	29	-7	-9	1	-3	3	-1	1	30	30	30	3	3	3	3	3	3	0
χ_{11}	546	154	26	2	14	6	-2	-2	0	0	-21	-12	51	6	6	-3	-3	0	3	0
χ_{12}	819	-21	-21	19	-1	7	-1	-1	1	1	90	9	9	9	9	9	9	0	0	0
χ_{13}	910	-210	30	-2	-10	2	2	-2	0	0	19	-17	55	-8	19	1	1	-2	4	-2
χ_{14}	910	-210	30	-2	-10	2	2	-2	0	0	19	-17	55	19	-8	1	1	-2	4	1
χ_{15}	1092	-140	52	4	0	4	4	0	0	0	-42	30	-6	-15	-15	3	12	0	-3	0
χ_{16}	1365	245	5	-27	5	1	1	-3	-1	-1	69	-3	60	15	15	6	6	0	3	0
χ_{17}	1365	-35	45	5	5	5	-3	-3	1	1	-12	87	-30	15	15	-3	-3	3	0	0
χ_{18}	1560	120	-40	-8	0	0	0	0	0	0	-60	12	30	-6	-6	3	3	0	-3	0
χ_{19}	1560	120	-40	-8	0	0	0	0	0	0	-60	12	30	-6	-6	3	3	0	-3	0
χ_{20}	1638	294	54	-10	6	2	2	-2	0	0	-63	18	45	-9	-9	0	9	0	0	0
χ_{21}	1820	140	-20	-4	0	4	4	0	0	0	-43	56	20	11	11	-7	-7	-1	5	-1
χ_{22}	2106	-414	66	-6	-14	6	-2	2	0	0	-81	0	81	0	0	0	0	0	0	0
χ_{23}	2184	56	24	-24	-4	0	0	-4	0	0	78	96	-21	24	24	6	6	3	0	0
χ_{24}	2457	189	21	33	-9	-7	5	3	-1	1	27	54	0	27	27	0	0	0	0	0
χ_{25}	2730	440	90	26	10	-2	-2	2	0	0	-24	-15	75	3	3	3	12	-3	0	0
χ_{26}	2730	-70	10	-6	-10	6	6	-2	0	0	138	21	-15	-24	-24	3	-6	0	-3	0
χ_{27}	2835	315	75	3	-5	3	-5	3	-1	-1	-81	81	0	0	0	0	0	0	0	0
χ_{28}	4095	315	-45	-25	5	11	-1	1	-1	1	207	18	45	-9	-9	9	0	0	0	0
χ_{29}	4095	-525	75	7	5	-9	3	1	1	-1	-36	72	45	18	18	0	9	0	0	0
χ_{30}	4368	560	48	16	0	0	0	0	0	0	156	-24	66	-6	-6	3	-6	-3	0	0
χ_{31}	4536	-504	-24	24	-4	0	0	-4	0	0	162	0	81	0	0	0	0	0	0	0
χ_{32}	5265	225	-15	33	5	-3	-3	-3	-1	-1	162	81	0	0	0	0	0	0	0	0
χ_{33}	5460	420	100	20	0	4	4	0	0	0	33	96	-30	-21	-21	-3	-3	0	-6	0
χ_{34}	5460	-700	100	-12	0	-4	-4	0	0	0	114	6	60	-21	-21	6	6	-3	-3	0
χ_{35}	5460	-140	-140	20	0	12	-4	0	0	0	-129	60	60	6	6	-3	15	-3	-3	0
χ_{36}	5824	896	64	0	16	0	0	0	0	0	-8	-8	154	-8	-8	-8	-8	1	1	1
χ_{37}	5824	-896	64	0	-16	0	0	0	0	0	-8	-8	154	-8	-8	-8	-8	1	1	1
χ_{38}	6552	-504	-24	-8	4	0	0	4	0	0	234	72	45	18	18	9	0	0	0	0
χ_{39}	7020	540	60	12	0	4	4	0	0	0	-27	-54	0	27	54	0	0	0	0	0
χ_{40}	7020	540	60	12	0	4	4	0	0	0	-27	-54	0	54	-27	0	0	0	0	0
χ_{41}	7280	-560	80	-16	0	0	0	0	0	0	-10	152	-10	-10	-10	-10	-10	-1	-1	-1
χ_{42}	7280	-560	80	-16	0	0	0	0	0	0	-10	-64	-10	44	-10	-1	8	-1	-1	-1
χ_{43}	7280	-560	80	-16	0	0	0	0	0	0	-10	-64	-10	-10	44	-1	8	-1	-1	2
χ_{44}	7371	819	51	-21	-1	-9	-1	-1	1	1	81	81	81	0	0	0	0	0	0	0
χ_{45}	8190	-210	30	-18	10	2	2	2	0	0	171	-45	-45	36	-45	0	-9	0	0	0
χ_{46}	8190	-210	30	-18	10	2	2	2	0	0	171	-45	-45	-45	36	0	-9	0	0	0
χ_{47}	11648	0	128	0	0	0	0	0	0	0	-16	-16	-124	-16	-16	2	20	2	2	-1
χ_{48}	14742	630	102	6	-10	-6	-6	-2	0	0	162	-81	-81	0	0	0	0	0	0	0
χ_{49}	16380	-420	60	28	0	12	-4	0	0	0	99	-36	-90	18	18	-9	-9	0	0	0
χ_{50}	16640	1280	0	0	0	0	0	0	0	0	-208	-64	80	8	8	8	-10	2	-4	-1
χ_{51}	16640	-1280	0	0	0	0	0	0	0	0	-208	-64	80	8	8	8	-10	2	-4	-1
χ_{52}	17472	-896	-64	0	16	0	0	0	0	0	-24	-24	30	-24	-24	-6	12	3	3	0
χ_{53}	17472	896	-64	0	-16	0	0	0	0	0	-24	-24	30	-24	-24	-6	12	3	3	0
χ_{54}	17920	0	0	0	0	0	0	0	0	0	-224	64	-80	-8	-8	10	-8	-2	4	1
χ_{55}	17920	0	0	0	0	0	0	0	0	0	-224	64	-80	-8	-8	10	-8	-2	4	1
χ_{56}	19683	-729	-81	27	9	-9	3	-3	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{57}	21840	560	-80	-48	0	0	0	0	0	0	-30	24	-30	24	24	-12	6	-3	-3	0
χ_{58}	22113	189	-171	9	-9	-3	1	3	1	-1	243	0	0	0	0	0	0	0	0	0

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
o(x)	9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	12	12	12
|C(x)|	3^3	$2^5 3^6$	$2^4 3^5$	$2^4 3^5$	$2^2 3^4$	$2^5 3^4$	$2^5 3^3$	$2^4 3^3$	$2^5 3^3$	$2^4 3^3$	$2^4 3^3$	$2^6 3^3$	$2^3 3^3$	$2^3 3^3$	$2\cdot3^3$	$2^2 3^3$	$2^2 3^3$	$2^3 3^2$	$2^3 3^2$	$2^3 3$
|cl(x)|	169,827,840	196,560	1,179,360	1,179,360	14,152,320	1,769,040	5,307,120	10,614,240	5,307,120	10,614,240	10,614,240	2,653,560	21,228,480	21,228,480	84,913,920	42,456,960	42,456,960	63,685,440	63,685,440	191,056,320
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	0	-7	2	-7	2	5	2	-1	1	1	-2	-2	1	1	1	-2	-2	0	-3	-1
χ_3	1	6	-3	-3	-3	2	-1	5	-2	1	1	-5	-2	-2	1	1	1	-1	-1	+1
χ_4	0	-8	-8	1	1	-4	2	-1	4	1	4	-2	1	1	-2	1	1	-2	1	-1
χ_5	0	2	2	11	2	6	0	3	2	-1	2	8	2	2	-1	2	2	-2	1	1
χ_6	-1	16	7	7	-2	4	7	1	0	3	3	3	-3	-3	0	0	0	1	1	-1
χ_7	0	1	10	1	1	-3	0	3	5	5	2	8	2	2	2	-1	-1	2	-1	1
χ_8	2	-7	2	2	2	-7	2	2	+1	-2	-2	-2	-2	1	1	4	-2	0	0	0
χ_9	-1	-7	2	2	2	-7	2	2	1	-2	-2	-2	1	-2	1	-2	4	0	0	0
χ_{10}	0	-10	-10	-10	-1	2	2	2	2	2	2	-10	-1	-1	-1	-1	-1	0	0	0
χ_{11}	0	19	-8	1	1	-1	8	-1	-1	5	-4	-4	2	2	-1	-1	-1	2	-1	1
χ_{12}	0	6	-3	-3	-3	6	-3	-3	10	1	1	1	1	1	1	1	1	-1	-1	-1
χ_{13}	1	-21	-3	-3	-3	3	3	3	-5	1	1	7	4	-5	1	1	1	-1	-1	1
χ_{14}	-2	-21	-3	-3	-3	3	3	3	-5	1	1	7	-5	4	1	1	1	-1	-1	1
χ_{15}	0	22	4	-14	4	-2	-2	-2	-2	-2	4	-2	1	1	1	-2	-2	0	0	0
χ_{16}	0	29	11	2	2	5	5	-4	-3	-6	3	-3	3	3	0	0	0	-1	2	0
χ_{17}	0	-8	1	-8	1	0	3	6	-4	-4	5	-1	-1	-1	-1	-1	-1	-1	2	0
χ_{18}	0	12	-6	-6	3	-4	8	2	4	-2	-2	4	-2	-2	1	1	1	0	0	0
χ_{19}	0	12	-6	-6	3	-4	8	2	4	-2	-2	4	-2	-2	1	1	1	0	0	0
χ_{20}	0	-3	-12	15	-3	9	-6	-3	5	-1	-4	2	-1	-1	2	-1	-1	0	3	1
χ_{21}	-1	5	-4	-4	5	-11	-8	4	5	-4	-4	8	-1	-1	-1	-1	-1	0	0	0
χ_{22}	0	-9	18	-9	0	3	0	-3	3	3	-6	0	0	0	0	0	0	-2	1	-1
χ_{23}	0	2	2	11	2	6	0	3	-6	3	-6	0	0	0	0	0	0	2	-1	-1
χ_{24}	0	27	0	0	0	3	6	0	3	0	0	6	3	3	0	0	0	0	0	0
χ_{25}	0	4	-5	13	4	0	-3	3	-4	5	-1	-7	-1	-1	-1	2	2	1	1	-1
χ_{26}	0	38	-7	11	2	10	1	1	6	3	-3	-3	0	0	-3	0	0	-1	-1	1
χ_{27}	0	-9	-9	18	0	3	-3	0	3	-6	3	9	0	0	0	0	0	1	-2	0
χ_{28}	0	-9	18	-9	0	-9	-6	-3	-1	-1	2	2	-1	-1	-1	2	2	2	-1	1
χ_{29}	0	-12	6	-21	-3	12	0	-3	4	-5	-2	-8	-2	-2	-2	1	1	2	-1	1
χ_{30}	0	20	20	2	2	12	0	6	4	-2	4	-8	-2	-2	1	-2	-2	0	0	0
χ_{31}	0	-18	-18	9	0	-6	0	-3	6	-3	6	0	0	0	0	0	0	2	-1	-1
χ_{32}	0	-18	9	-18	0	-6	-3	0	6	6	-3	9	0	0	0	0	0	-1	2	0
χ_{33}	0	-39	6	6	-3	1	4	-2	-7	2	2	8	-1	-1	-1	-1	-1	0	0	0
χ_{34}	0	2	-16	-16	2	10	-2	4	-6	0	0	6	3	3	0	0	0	0	0	0
χ_{35}	0	-5	4	4	-5	-5	4	4	-1	-4	-4	-4	2	2	-1	-1	-1	0	0	0
χ_{36}	1	32	-4	-4	-4	-8	4	-2	0	0	0	0	0	0	0	0	0	-2	-2	0
χ_{37}	1	-32	4	4	4	-8	4	-2	0	0	0	0	0	0	0	0	0	2	2	0
χ_{38}	0	-18	-18	9	0	-6	0	-3	-2	1	-2	-8	-2	-2	1	-2	-2	-2	1	1
χ_{39}	0	-27	0	0	0	-3	-6	0	-3	0	0	-6	-3	6	0	0	0	0	0	0
χ_{40}	0	-27	0	0	0	-3	-6	0	-3	0	0	-6	6	-3	0	0	0	0	0	0
χ_{41}	-1	34	-2	-2	-2	-10	-4	2	2	2	2	-16	2	2	2	2	2	0	0	0
χ_{42}	2	34	-2	-2	-2	-10	-4	2	2	2	2	8	-4	2	-1	-4	2	0	0	0
χ_{43}	-1	34	-2	-2	-2	-10	-4	2	2	2	2	8	2	-4	-1	2	-4	0	0	0
χ_{44}	0	9	9	9	0	-3	-3	-3	-3	-3	-3	9	0	0	0	0	0	-1	-1	-1
χ_{45}	0	-21	-3	-3	-3	3	3	3	3	-3	-3	3	0	3	0	3	-3	1	1	-1
χ_{46}	0	-21	-3	-3	-3	3	3	3	3	-3	-3	3	3	0	0	-3	3	1	1	-1
χ_{47}	-1	0	0	0	0	-16	8	-4	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	-18	9	9	0	-6	3	3	6	-3	-3	-9	0	0	0	0	0	-1	-1	1
χ_{49}	0	39	-6	-6	3	15	0	-6	-5	-2	-2	4	-2	-2	1	1	1	0	0	0
χ_{50}	-1	-16	-16	-16	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	-1	16	16	16	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	-32	4	4	4	8	-4	2	0	0	0	0	0	0	0	0	0	-2	-2	0
χ_{53}	0	32	-4	-4	-4	8	-4	2	0	0	0	0	0	0	0	0	0	2	2	0
χ_{54}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	-34	2	2	2	10	4	-2	6	6	6	0	0	0	0	0	0	0	0	0
χ_{58}	0	27	0	0	0	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	0

	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
o(x)	12	12	12	12	12	18	18	18	18	5	10	10	20	15	7	14	13	13
|C(x)|	$2^4 3^2$	$2^4 3^2$	$2^2 3^2$	$2^2 3^2$	$2^4 3$	$2^2 3^3$	$2^2 3^3$	$2\cdot3^3$	$2^2 3^2$	$2^3\cdot3\cdot5$	$2^3\cdot5$	$2^2\cdot5$	$2^2\cdot5$	$3\cdot5$	$2\cdot7$	$2\cdot7$	13	13
|cl(x)|	31,842,720	31,842,720	127,370,880	127,370,880	95,528,160	42,456,960	42,456,960	84,913,920	127,370,880	38,211,264	114,633,792	229,267,584	229,267,584	305,690,112	327,525,120	327,525,120	352,719,360	352,719,360
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	2	-1	-1	-1	-1	-1	-1	2	-1	3	-1	1	-1	0	1	1	0	0
χ_3	3	0	0	0	0	0	0	0	2	1	1	-1	-1	1	0	0	0	0
χ_4	2	2	-1	-1	-2	1	1	-2	-1	0	0	0	0	0	0	0	1	1
χ_5	0	0	0	0	0	2	2	-1	0	3	-1	1	-1	0	0	0	-1	-1
χ_6	-1	2	-1	-1	2	1	1	1	1	2	2	0	0	-1	0	0	0	0
χ_7	0	3	0	0	-1	-2	-2	1	0	0	0	0	0	0	-1	-1	0	0
χ_8	-2	1	-2	1	1	-1	-1	-1	-1	0	0	0	0	0	1	-1	0	0
χ_9	-2	1	1	-2	1	-1	-1	-1	-1	0	0	0	0	0	1	-1	0	0
χ_{10}	-2	4	1	1	0	-1	-1	-1	-1	3	-1	-1	1	0	0	0	0	0
χ_{11}	0	-3	0	0	1	1	1	-2	-1	1	1	-1	-1	1	0	0	0	0
χ_{12}	1	-2	1	1	2	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0
χ_{13}	-1	-1	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	-1	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	-2	-2	1	1	-2	1	1	-2	1	2	2	0	0	-1	0	0	0	0
χ_{16}	1	1	1	1	1	-1	-1	2	-1	0	0	0	0	0	0	0	0	0
χ_{17}	-1	-4	-1	-1	0	-2	-2	1	0	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	$3+6\omega$	$-3-6\omega$	0	-1	0	0	0	0	0	-1	1	0	0
χ_{19}	0	0	0	0	0	$-3-6\omega$	$3+6\omega$	0	-1	0	0	0	0	0	-1	1	0	0
χ_{20}	2	-1	-1	-1	-1	0	0	0	0	+3	-1	-1	1	0	0	0	0	0
χ_{21}	4	1	1	1	1	-1	-1	-1	1	0	0	0	0	0	0	0	0	0
χ_{22}	0	-3	0	0	1	0	0	0	0	1	1	1	1	1	-1	-1	0	0
χ_{23}	0	0	0	0	0	2	2	-1	0	-1	-1	1	1	-1	0	0	0	0
χ_{24}	2	-1	-1	-1	-1	0	0	0	0	-3	1	-1	1	0	0	0	0	0
χ_{25}	1	-2	1	1	-2	-2	-2	1	0	0	0	0	0	0	0	0	0	0
χ_{26}	-3	0	0	0	0	-1	-1	2	1	0	0	0	0	0	0	0	0	0
χ_{27}	-3	-3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1
χ_{28}	2	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	2	2	-1	0	-2	-2	0	0	1	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	-1	-1
χ_{32}	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
χ_{33}	4	1	1	1	1	0	0	0	-2	0	0	0	0	0	0	0	0	0
χ_{34}	2	2	-1	-1	2	-1	-1	-1	1	0	0	0	0	0	0	0	0	0
χ_{35}	0	3	0	0	-1	1	1	1	1	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	-1	-1	-1	1	-1	-1	1	1	-1	0	0	0	0
χ_{37}	0	0	0	0	0	1	1	1	1	-1	-1	-1	-1	-1	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	-3	1	1	-1	0	0	0	0	0
χ_{39}	-2	1	1	-2	1	0	0	0	0	0	0	0	0	0	-1	1	0	0
χ_{40}	-2	1	-2	1	1	0	0	0	0	0	0	0	0	0	-1	1	0	0
χ_{41}	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0
χ_{44}	-3	3	0	0	-1	0	0	0	0	1	1	-1	-1	1	0	0	0	0
χ_{45}	-1	-1	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	-1	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	2	-2	-2	0	0	1	0	0	0	0
χ_{48}	3	0	0	0	0	0	0	0	0	2	2	0	0	-1	0	0	0	0
χ_{49}	0	3	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	2	2	2	0	0	0	0	0	0	1	-1	0	0
χ_{51}	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	1	1	0	0
χ_{52}	0	0	0	0	0	1	1	1	-1	-3	1	-1	1	0	0	0	0	0
χ_{53}	0	0	0	0	0	-1	-1	-1	-1	-3	1	1	-1	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	α	$-\alpha-1$
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha-1$	α
χ_{56}	0	0	0	0	0	0	0	0	0	3	-1	1	-1	0	-1	-1	1	1
χ_{57}	0	0	0	0	0	-1	-1	-1	1	0	0	0	0	0	0	0	0	0
χ_{58}	0	-3	0	0	1	0	0	0	0	3	-1	-1	1	0	0	0	0	0

CHARACTER TABLE
OF O(7,3) OF
ORDER = 4,585,351,680

$\omega^3 = 1$
$\alpha^2+\alpha = 3$

§16 The Conjugacy Classes of M(22)

From Theorem 1.2 we know that M(22) contains a unique conjugacy class of 3 transpositions denoted by D.

Let π be a fixed element in D. Then $C_{M(22)}(\pi)/\langle\pi\rangle \cong$ PSU(6,2). We denote $C_{M(22)}(\pi)$ by P*. $|P^*| = 2^{16}.3^6.5.7.11$.

Suppose $\chi^{(22)}$ is a character of M(22) and that $x \in P^* \leq M(22)$. Then we denote $x\{1,\pi\}$ by $\tilde{x}$ and regard $\tilde{x}$ as an element of PSU(6,2). Then $\bar{\chi}^{(22)}(\tilde{x}) = \dfrac{\chi^{(22)}(x) + \chi^{(22)}(x\pi)}{2}$ is a character of PSU(6,2).

Now we know that M(22) has 2 rank 3 representations of degrees 3510 and 14080 (Lemma 1.4). The conjugacy classes of M(22) are found by using the characters of these representations and their restrictions to the subgroups P* and P$S\Omega$(7,3).

Conjugacy classes of involutions in M(22)

We remind the reader that T and N denote conjugacy classes of involutions consisting of products of commuting pairs and triples of elements from D.

$|D| = 3510 = 2.3^3.5.13$

$|D_\pi| = 693 = 3^2.7.11$ (D_π is the set of elements in D $\neq \pi$ which commute with π, the number 693 comes from TABLE 5.)

$\therefore\ |T| = 693.3510/2$ (Lemma 1.)

$= 3^5.5.7.11.13$

$\therefore\ |C_{M(22)}(t)| = 2^{17}.3^4.5$ (We note that t is a central involution in its Sylow 2-group.)

Let d be an involution in PSU(6,2). Then $|D_d| = 180$ $\therefore$. The number of commuting triples of involutions in M(22) is 3510.693.180/3.2.1. But we know from §3 that each element in N can be represented as the product of commuting triples of elements from D is precisely 2 ways.

$\therefore |N| = 2.3^6.5^2.7.11.13$ and $|C_{M(22)}(n)| = 2^{16}.3^3$.

These 3 conjugacy classes of involutions are all the involutions in M(22). For suppose $x^2=1\neq x$, $x \notin D,T,N$. $x \in S \in Syl_2(G)$. $\therefore S \subset N_{M(22)}(L_{10})$ for some L_{10}. $|L_{10}| = 2^{10}$. (See §3.) We know $x \notin L_{10}$ as all elements in L_{10} are 1 or in D,T and N. $\therefore$ xL is a nontrivial element of M_{22}. All involutions in M_{22} are of the form $1^6 2^8$ as permutations on 22 symbols. That is they all centralize an element in D. Hence $x \in P^*$. But all elements in P* are in D,T or N.

o(x)	1	2	2	2
rep'tive	1	d	t	n
$\lvert C_{M(22)}(x)\rvert$	$2^{17}.3^9.5^2.$ 7.11.13	$2^{16}.3^6$ 5.7.11	$2^{17}.3^4.5$	$2^{16}.3^3$
$\chi^{(22)}_{3510}$	3510	694	182	54
$\chi^{(22)}_{14080}$	14080	1408	256	128
# in P*	1	2,1π	3,2π	3π,4*
# in PSΩ(7,3)	1	2	3	4
# in M(22)	1	2	3	4

In the above table 1 and 1π are 2 conjugacy classes in P* which come from conjugacy class number 1 in P. 4* is the unique conjugacy class containing the elements from #4 in P.

$PS\Omega(7,3)$ acts on the 3510 cosets of P^* in orbits of length 3159 and 351. Hence the character $\chi^{(22)}_{3510}$ is immediately known for all conjugacy classes in $M(22)$ which have representatives in $PS\Omega(7,3)$.

We restrict $\chi^{(22)}_{3510}$ to P^* and calculate the associated character of $PSU(6,2)$.

$$\frac{3510 + 694}{2} = 2102 \qquad \frac{694 + 182}{2} = 438 \qquad \frac{182 + 54}{2} = 118$$

$\dfrac{54 + 54}{2} = 54$ are the values of $\bar{\chi}^{(22)}_{3510}$ on 1, D, T and N.

Thus $\bar{\chi}^{(22)}_{3510} = 3\chi^{(6)}_{1} + 2\chi^{(6)}_{4} + \chi^{(6)}_{6} + \chi^{(6)}_{11}$. Hence $\bar{\chi}^{(22)}_{3510}(x) = (\chi^{(22)}_{3510}(x) + \chi^{(22)}_{3510}(\pi x))/2$ can be calculated for all $x \in P^*$.

<u>Conjugacy classes of elements of order divisible by 13,11,7,5</u>

The centralizer of an element of order 13, f, is a 3-group. $|N_{M(22)}(\langle f\rangle)/C_{M(22)}(\langle f\rangle)| = 6$ or 12. (See $PS\Omega(7,3)$). $\therefore |N_{M(22)}(\langle f\rangle)| = 13.2^a.3^b$. By Sylow's theorem the order is $13.2.3$ or $13.2.3^4$ or $13.2.3^7$

Later we shall prove that all elements of order 3 do not centralize an element of order 13. $\therefore |C_{M(22)}(f)| = 13$ and there are ~~X~~ 2 conjugacy classes of elements of order 13.

Let f be an element of order 11 in $M(22)$. f lies in P^* and $\bar{\chi}^{(22)}_{3510}(f) = 1$. $\therefore$ elements of order 11 and 22 both centralize exactly one $d \in D$. Hence $C_{M(22)}(f) = C_{P^*}(f)$. By Sylow's theorem $|N_{M(22)}(\langle f\rangle)| = 2.5.11$. $\therefore$ there exists 2 conjugacy classes of elements of order 11 and 2 conjugacy classes of elements of order 22 in $M(22)$.

Let f be an element of order 7. $\chi^{(22)}_{3510}(f) = 3$ and $\chi^{(22)}_{3510}(\pi f) = 1$. $\therefore$ $|C_{M(22)}(\pi f)| = 2.7$ as any element which centralizes πf, centralizes π and hence ε P*. Also $C_{M(22)}(f) = \Sigma_3 \times C_7$.

There is 1 conjugacy class of elements of order 7, centralizer of order 2.3.7; f.

There is 1 conjugacy class of elements of order 14, centralizer of order 2.7; df.

There is 1 conjugacy class of elements of order 21, centralizer of order 3.7; αf.

(α denotes the product of 2 non-commuting elements in D.)

B.Fischer has shown that there is only one conjugacy class of elements of order 5 in M(22), M(23) and M(24) by considering a subgroup which contains $W(E_8)$ of index 6. However, it will be clear by the end of this section that any element of order 5 other than that discussed below must be self-centralizing and there will not be enough elements left by then.

Hence we consider the element of order 5 which centralize an element in D, f. M(22) contains subgroups isomorphic to Σ_{10} (Lemma 1.4.viii) and $C_{\Sigma_{10}}(f) \cong \Sigma_5 \times C_5$. $\chi^{(22)}_{3510}(f) = 10$. Any element centralizing f permutes these 10 elements in D. Now Σ_5 permutes the 10 elements in $D \cap \Sigma_5$ transitively and also $C_{P*}(f) = C_5 \times \Sigma_3 \times C_2 \leq \Sigma_5 \times C_5$. $\therefore$ $C_{M(22)}(f) \cong \Sigma_5 \times C_5$.

There is 1 conjugacy class of elements of order 5, centralizer of order $2^3.3.5$; f.

There is one conjugacy class of elements of order 10, centralizer of order $2^2.3.5$; df.

There is 1 conjugacy class of elements of order 10, centralizer of order $2^2.5$; tf.

There is 1 conjugacy class of elements of order 15, centralizer of order 2.3.5; αf.

There is 1 conjugacy class of elements of order 20, centralizer of order $2^2.5$; vf. (v is an element of order 4 whose square is in T.)

There is 1 conjugacy class of elements of order 30, centralizer of order 2.3.5; αdf.

o(x)	13	13	11	11	22	22	7	14
rep'tive								
$\lvert C_{M(22)}(x)\rvert$	13	13	2.11	2.11	2.11	2.11	2.3.7	2.7
$\chi^{(22)}_{3510}$	0	0	1	1	1	1	3	1
$\chi^{(22)}_{14080}$	1	1	0	0	0	0	3	1
# in P*			44	45	44π	45π	46	46π
# in PSΩ(7,3)	56	57					54	55
# in M(22)	5	6	7	8	9	10	11	12

o(x)	21	5	10	10	15	20	30
rep'tive		f	df	tf	αf	vf	αdf
$\lvert C_{M(22)}(x)\rvert$	3.7	$2^3.3.5^2$	$2^2.3.5$	$2^3.5$	2.3.5	$2^2.5$	2.3.5
$\chi^{(22)}_{3510}$	0	10	4	2	1	0	1
$\chi^{(22)}_{14080}$	0	5	3	1	2	1	0
# in P*		42	42π 41	41π	43		43π
# in PSΩ(7,3)		49	51	50	53	52	
# in M(22)	13	14	15	16	17	18	19

Conjugacy classes of elements of order divisible by 3

The order of a Sylow 3-subgroup of $M(22)$ is the same as that of $PS\Omega(7,3)$. $\therefore$ every element of order a power of 3 is represented in a conjugacy class of $PS\Omega(7,3)$.

αx x an element of order 2^a

Let $d' \in A_d$ and let $\alpha = dd'$. $M(22)$ acts as a rank 3 group on the conjugacy class D (Lemma 1.4). Therefore all elements of the same form as α are conjugate in $M(22)$. Therefore α has $\frac{3510.2816}{3} = 2^9.3^2.5.11.13$ conjugates. And $|C_{M(22)}(\alpha)| = 2^8.3^7.5.7$. Now if 2 elements αx and αy, $x,y \in C_{M(22)}(\alpha)$, $|x| = 2^a$ are conjugate then they are conjugate in $C_{M(22)}(\alpha)$, indeed x,y are conjugate in $C_{M(22)}(\alpha) = C_3 \times P\Omega^-(6,3)$. Hence we get the following conjugacy classes.

$o(x)$	3	6	6	6	12
rep'tive	α	αd	αt	αn	αq
$\lvert C_{M(22)}(x)\rvert$	$2^8.3^7.5.7$	$2^7.3^5.5$	$2^8.3^3$	$2^7.3^3$	$2^5.3^2$
$\chi^{(22)}_{3510}$	126	46	14	6	6
$\chi^{(22)}_{14080}$	112	40	16	8	4
# in P*	16	16π 17	17π 18	18π	20
# in $PS\Omega(7,3)$	13	24	28	30	38
# in $M(22)$	20	21	22	23	24

o(x)	12	12	12	24	24
rep'tive	αq	αdv	αdq		
$\lvert C_{M(22)}(x)\rvert$	$2^6.3^2$	$2^5.3$	$2^6.3^2$	$2^4.3$	$2^4.3$
$\chi^{(22)}_{3510}$	2	2	2	0	0
$\chi^{(22)}_{14080}$	0	4	0	0	0
# in P*	19	20π	19π		
# in PSΩ(7,3)		39			
# in M(22)	25	26	27	28	29

There are 7 conjugacy classes of elements of order 3 in PSΩ(7,3).

#	$\lvert C_{PS\Omega(7,3)}(x)\rvert$	$\chi^{(22)}_{3510}$
11	$2^5.3^9$	27
12	$2^6.3^7$	36
13	$2^4.3^7.5$	126
14	$2^3.3^7$	27
15	$2^3.3^7$	0
16	2.3^6	0
17	$2^2.3^6$	36

There are 3 conjugacy classes of elements of order 3 in P*.

#	$\lvert C_{P*}(x)\rvert$	$(\chi^{(22)}_{3510}(x)+\chi^{(22)}_{3510}(x\pi))/2$
16	$2^7.3^5.5$	86
21	$2^6.3^6$	23
35	$2^3.3^5$	23

As elements in $PS\Omega(7,3)$ take 3 values 27, 36 and 126 the 3 classes in P* are not fused at all in M(22). As #11 in $PS\Omega(7,3)$ and 21 in P* are both central they fuse in M(22).

Hence #21 in P* fuses to #11 and #14 in $PS\Omega(7,3)$.

#16 in P* fuses to #13 in $PS\Omega(7,3)$.

#35 in P* fuses to #12 and #17 in $PS\Omega(7,3)$.

It is now possible to calculate $\chi^{(22)}_{14080}$ on all elements represented in $PS\Omega(7,3)$ using all fusions already determined.

ζx, x an element of order 2^a

Let ζ be an element of order 3 central in the Sylow 3-subgroup of M(22). $\chi^{(22)}_{14080}(\zeta) = 148$.

$\therefore\ |C_{M(22)}(\zeta)| = 2^5.3^9.148/(1+36) = 2^7.3^9.$

$|C_D(\zeta)|\ (=\chi^{(22)}_{3510}(\zeta)) = 27 \equiv 1 \pmod 2$

$\therefore$ if $o(x) = 2^a$, $x \in C_{M(22)}(\zeta)$ then $|C_D(x\zeta)| \not\equiv 0$,

i.e. $x\zeta \in P^*$.

We list the relevant elements of order 6 and 12 in $PS\Omega(7,3)$.

#	$o(\zeta x)$	$\chi^{(22)}_{3510}(\zeta x)$	$\chi^{(22)}_{14080}(\zeta x)$	$\lvert C_{PS\Omega(7,3)}(\zeta x)\rvert$
22	6	19	4	$2^5.3^6$
26	6	11	4	$2^5.3^4$
29	6	20	3	$2^5.3^3$
33	6	20	3	$2^3.3^3$
41	12	3	4	$2^4.3^2$
42	12	3	4	$2^2.3^2$
44	12	3	4	$2^4.3$

Clearly #29 fuses to #33 in M(22). #22 is ζd, #26 is ζt and #29 and #33 are of the form ζn. The elements in P* fusing to the 3 conjugacy classes of elements of order 6 are clear, and these exhaust all conjugacy classes of order 6 in P* related to ζ.

The conjugacy classes of order 12 related to ζ in PSU(6,2) are :=

#	$\lvert C_{PSU(6,2)}(\zeta\bar{x})\rvert$	$\bar{\chi}^{(22)}_{3510}(\tilde{\zeta}\tilde{x})$
25	$2^5.3^3$	11
26	$2^5.3^3$	11
27	$2^5.3$	3
28	$2^5.3$	3
29	$2^4.3$	7
30	$2^4.3$	3
31	$2^4.3$	3

Now if $\lvert\zeta x\rvert = 12$ then (**) $\lvert Syl_2(C_{M(22)}(\zeta x))\rvert = 2^4$ or 2^5. (**) will be proved at the end of this subsection.

$\therefore$ In P* for n=5,6,7,8 #2n is fused to #2nπ.

Also by (**) #41 and #44 in PSΩ(7,3) fuse to one conjugacy class in M(22) #36 = $\{p_1\zeta\}$.

$\lvert C_{M(22)}(p_1\zeta)\rvert = 2^4.3^2$.

It is now clear that #30 fuses to #30π and #31 fuses to #31π in P* and also that they fuse to #41,#44 and #42 in PSΩ(7,3) respectively.

Now either #29 or #29π must fuse to #25* and #26* and the other fuse to #27* and #28*. Hence we have the following conjugacy classes.

o(x)	3	6	6	6
rep'tive	ζ	dζ	tζ	nζ
$\lvert C_{M(22)}(x)\rvert$	$2^7.3^9$	$2^7.3^6$	$2^7.3^4$	$2^7.3^3$
$\chi^{(22)}_{3510}$	27	19	11	3
$\chi^{(22)}_{14080}$	148	4	4	20
# in P*	21	21π 22 23	22π 23π 24	24π
# in PSΩ(7,3)	11 14	22	26	29 33
# in M(22)	30	31	32	33

o(x)	12	12	12	12
rep'tive	$q_1\zeta$	$q_2\zeta$	$p_1\zeta$	$p_2\zeta$
$\lvert C_{M(22)}(x)\rvert$	$2^5.3^3$	$2^5.3$	$2^4.3^2$	$2^4.3^2$
$\chi^{(22)}_{3510}$	11	3	3	3
$\chi^{(22)}_{14080}$	0	0	4	4
# in P*	25* 26* 29	27* 28* 29π	30*	31*
# in PSΩ(7,3)			41 44	42
# in M(22)	34	35	36	37

<u>Proof (**)</u> This proof has arisen out of discussions with B. Fischer. No attempt is made to justify the facts given.

Let H be the group generated by 3 involutions $a,b,c \in D$ none of which commute with each other and satisfying $(a^b)^c \neq a^b$.

$|H \cap D| = 9,$

$|H| = 54, \quad |Z(H)| = 3, \quad |H'| = 27$ and

H' is extra-special of exponent 3.

$|C_D(\zeta)| = 27 = 3.9$

$\langle C_D(\zeta)\rangle \cong H_1 \curlyvee H_2 \curlyvee H_3$ of order $2^3.3^7$ the central product of 3 H's.

Let d_1, d_2, d_3 be elements of D from H_1, H_2, H_3. Then $\langle d_1, d_2, d_3\rangle$ is elementary abelian of order 8 and lies in the center of a Sylow 2-subgroup of $C_{M(22)}(\zeta)$ which we denote by S. For $i=1,2,3$ $S_i = S/\langle d_i\rangle \cong Q_{i_1} \times Q_{i_2}$ the direct product of 2 quaternion groups whose involutions are t's in S but d's in S_i.

Now we see that for any element x of order 4 $|Syl_2(C_{M(22)}(x\zeta))| \geq 2^4$ as $|Z(S)| = 2^3$. Now suppose $|C_S(x)| = 2^6$, then $|C_{S_i}(x)| = 2^5$ $i=1,2,3$. The involution x^2 is $d_1d_2, \ldots\ldots$ or $d_1d_2d_3$ (as all d's in D are non-squares see §3). Suppose $x^2 = d_1d_2$ then $|C_{S_3}(x)| = 2^4$, if $x^2 = d_1d_2d_3$ then $C_{S_i}(x) = 2^4$ for all i. Hence we have the desired conclusion $|Syl_2(C_{M(22)}(x\zeta))| \leq 2^5$. ##

Elements of order 9 and 18

All elements of order 9 are represented in $PS\Omega(7,3)$. Those whose cube fuse to ζ are listed below.

#	$\lvert C_{PS\Omega(7,3)}(x)\rvert$	$\chi^{(22)}_{3510}(x)$	$\chi^{(22)}_{14080}(x)$
18	2.3^4	3	7
19	$2^2.3^4$	6	1
20	3^3	3	7

Clearly #18 fuses to #20. In PSU(6,2) there are 3 conjugacy classes of elements of order 9:=

#	$\lvert C_{PSU(6,2)}(\bar{x})\rvert$	$\bar{\chi}^{(22)}_{3510}(\bar{x})$
36	$.3^3$	2
39	$2\ .3^3$	5
40	$2\ .3^3$	5

The fusion is obvious. As $\chi^{(22)}_{3510}(x)$ is odd for all elements x of order 9 whose cube is in ζ every element of order 18 whose 6th power is in ζ is represented in P*. There are 4 conjugacy classes of elements of order 18 in PSΩ(7,3) whose 6th powers are conjugate to ζ :=

#	$\lvert C_{PS\Omega(7,3)}(x)\rvert$	$\chi^{(22)}_{14080}(x)$	$\chi^{(22)}_{3510}(x)$
45	$2^2.3^3$	1	1
48	$2^2.3^3$	1	4
46	2.3^3	1	4
47	$2^2.3^3$	1	2

Hence there are 4 conjugacy classes of elements of order 18 whose 6th power lie in ζ.

o(x)	9	18	9	18	18	18
rep'tive	ν	$d\nu$	μ	$d\mu$	$d\mu$	$t\mu$
$\lvert C_{M(22)}(x)\rvert$	2.3^4	2.3^3	$2^2.3^4$	$2^2.3^3$	$2^2.3^3$	$2^2.3^3$
$\chi^{(22)}_{3510}(x)$	3	1	6	4	4	2
$\chi^{(22)}_{14080}(x)$	7	1	1	1	1	1
# in P*	36	36π	39 40	39π 37	40π 38	37π 38π
# in PSΩ(7,3)	18 20	46	19	45	48	46
# in M(22)	38	39	40	41	42	43

There are no elements of order 27 in M(22) as there are none in PSΩ(7,3). There is one further class of elements of order 9 discussed in a later subsection.

ρx, x an element of order 2^a

Conjugacy classes #12 and #17 in PSΩ(7,3) fuse to one conjugacy class with representative ρ in M(22). $|C_{M(22)}(\rho)| = |C_{PS\Omega(7,3)}(\rho)| = 2^6.3^7$ for those ρ which lie in conjugacy class #12 in PSΩ(7,3). Therefore every element of the form ρx, x an element of order 2^a is represented in PSΩ(7,3) :=

o(x)	#	$\lvert C_{PS\Omega(7,3)}(x)\rvert$	$\chi^{(22)}_{3510}(x)$	$\chi^{(22)}_{14080}(x)$
3	12	$2^6.3^7$	36	49
3	17	$2^2.3^6$	36	49
6	23	$2^4.3^5$	10	13
6	25	$2^2.3^4$	10	13
6	27	$2^5.3^3$	8	1

$o(x)$	#	$\lvert C_{PS\Omega(7,3)}(x)\rvert$	$\chi^{(22)}_{3510}(x)$	$\chi^{(22)}_{14080}(x)$
6	31	$2^4.3^3$	6	5
6	32	$2^6.3^3$	12	17
6	36	$2^2.3^3$	6	5
6	58	$2^2.3^3$	12	17
12	37	$2^3.3^2$	0	1
12	40	$2^4.3^2$	0	1

The conjugacy classes in PSU(6,2) whose powers lie in ρ are #35, #34, #33 and #32.

$o(x)$	6	6	6	6	12	12
rep'tive	$d\rho$	$t\rho$	$n\rho$	$n'\rho$	$q_3\rho$	$p_4\rho$
$\lvert C_{M(22)}(x)\rvert$	$2^4.3^5$	$2^5.3^3$	$2^6.3^3$	$2^4.3^3$	$2^3.3^2$	$2^4.3^2$
$\chi^{(22)}_{3510}$	10	8	12	6	0	0
$\chi^{(22)}_{14080}$	13	1	17	5	1	1
# in P*	35π 34	34π 33	32	32π 33π		
# in $PS\Omega(7,3)$	23 25	27	32 58	31 36	37	43
# in M(22)	49	50	51	52	53	54

<u>βx, x an element of order 2</u>

There are 2 remaining conjugacy classes of order 3 in $PS\Omega(7,3)$. We list all the conjugacy classes of $PS\Omega(7,3)$ some power of which lie in these 2 classes.

It is clear from the values of $\chi^{(22)}_{14080}$ on the elements of order 6 that these must fuse together in M(22) and hence that the elements of order 3 fuse too, to a conjugacy class with representative β.

#	$o(\beta x)$	$\lvert C_{PS\Omega(7,3)}(\beta x)\rvert$	$\chi^{(22)}_{3510}(\beta x)$	$\chi^{(22)}_{14080}(\beta x)$
15	3	$2^3.3^7$	0	13
16	3	2.3^6	0	13
34	6	$2^3.3^3$	0	5
35	6	2.3^3	0	5
43	12	$2^2.3^{\cancel{3}2}$	0	1
21	9	3^3	0	1

The above elements yield the following conjugacy classes in M(22).

$o(x)$	3	6	12	9
rep'tive	β	$n\beta$	$p_3\beta$	
$\lvert C_{M(22)}(x)\rvert$	$2^3.3^7$	$2^3.3^3$	$2^2.3^{\cancel{3}2}$	3^3
$\chi^{(22)}_{3510}$	0	0	0	0
$\chi^{(22)}_{14080}$	13	5	1	1
# in P*				
# in $PS\Omega(7,3)$	15 16	34 35	43	21
# in M(22)	44	45	46	47

Hence the list of conjugacy classes is complete except for elements of order 4,8,16,... .

<u>Elements of order 2^a</u>

There are 4 conjugacy classes of elements of order 4 in $PS\Omega(7,3)$:=

#	$\lvert C_{PS\Omega(7,3)}(x)\rvert$	$\chi^{(22)}_{3510}(x)$	$\chi^{(22)}_{14080}(x)$
5	$2^6.3^2.5$	30	16
6	$2^7.3^2$	6	16
7	$2^7.3$	6	16
8	$2^6.3$	14	16

Now clearly there are 2 conjugacy classes of elements of order 4.

# M(22)	$\lvert C_{M(22)}(x)\rvert$	$\chi^{(22)}_{3510}(x)$	
55	$2^{10}.3^2.5$	30	(α)
56	$2^{10}.3$	14	(β)

Also there are either 2 classes from #6 and #7 or they fuse together.

Now #7 does not fuse to #7π in P* as $\overline{\chi}^{(22)}_{3510}$ takes the value 22 on class #7.

Suppose that conjugacy class #7 in P* fuses to (α) then there exist other classes contributing 15 to $\chi^{(22)}_{3510}(\alpha)$ but inspection of the other classes shows that this is impossible. $\therefore$ #10 in P* fuses to (α) and #10π fuses to some conjugacy class with $\chi^{(22)}_{3510}$ equal to 14. Conjugacy classes which may fuse to (β) are #7π,#10π,#11,#11π or #11* which would contribute 1,2,6,6 and 12 respectively to (β). $\therefore$ 10π and either 11 and 11π or 11* fuse to (β).

We now have enough values of $\chi^{(22)}_{14080}$ to calculate $\frac{\chi^{(22)}_{14080}(x) + \chi^{(22)}_{14080}(x\pi)}{2}$ for all x in P*. This is zero for all elements of order 4 except conjugacy classes #8, #9, #10 and #11 in PSU(6,2) for each of which the value is 16. Hence #8 fuses to #8π and #9 fuses to #9π in P* and all fuse to #6 and #7 in PSΩ(7,3) to form conjugacy class #57 in M(22) with centralizer $2^9.3^2$.

There are 2 conjugacy classes of order 8 in PSΩ(7,3) and these fuse to conjugacy classes 14* and 15* respectively in P*. These either remain as 2 classes in M(22) #58a and #58b or they fuse to 1 class #58.

The other conjugacy classes of elements of order 4 contain no representatives in PSΩ(7,3). In PSU(6,2) :=

#	$\lvert C_{PSU(6,2)}(\bar{x})\rvert$	$\bar{\chi}^{(22)}_{3510}(\bar{x})$
5	$2^{11}.3^3$	38
6	$2^{11}.3$	6
7	$2^9.3$	22

Either 7 or 7π fuses to 5 or 5π and as $\lvert 2^{11}.3^3:2^9.3\rvert = 36$ we must have 5,5π and 7 all fuse to one conjugacy class #59 with centralizer $2^{12}.3^3$ and character 38. (Possibly 5 and 5π fuse to one conjugacy class 5* in P* but this does not affect the conjugacy classes in M(22).) Either 6,6π and 7 fuse to one conjugacy class in M(22) or they yield 2 or 3 classes in M(22). However, a check of the number of conjugacy classes of order 12 shows they fuse to one class.

Now in PSU(6,2) the remaining elements of order 8 are :=

#	$\|C_{PSU(6,2)}(\tilde{x})\|$	$\bar{\chi}^{(22)}_{3510}(\tilde{x})$
12	2^5	6
13	2^6	2

Clearly neither 12 nor 12π fuses to 13 or 13π. Hence from these elements we get either 2, 3 or 4 more conjugacy classes in M(22).

All the other conjugacy classes in M(22) are elements of order 2^a which do not centralize any element in D. A check of the elements already accounted for leaves exactly 1/16 of the elements in the group. Also we have accounted for all elements which centralize an element of order prime to 2.

Let x be such an element not yet determined. Let S be a Sylow 2-subgroup of M(22) containing $C_{M(22)}(x)$. Then $S \leq N_{M(22)}(L_{10})$ for some elementary abelian subgroup L_{10}. (See §3). $\therefore$ in $N_{M(22)}(L_{10})/L_{10}$, xL_{10} is an element of order 8 in M(22). x acts as a permutation $2^1 4^1 8^2$ on the 22 elements of D contained in L_{10}, where the $2^1 4^1$ form a hexad. $\therefore$ $|C_{L_{10}}(x)| = 4$ for if the hexad is d_1, d_2, d_3, d_4, d_5 and d_6 then x centralizes 1, $d_1 d_2$, $d_1 d_3 d_5$ and $d_1 d_4 d_6$ where x acts as (12)(3456) on the hexad.

$|C_{M(22)}(xL)| = 8$ and $|C_{M(22)}(x)| = 32$. Clearly x is of order 8 or 16. However by considering the elements in N_{22} (See section 3) it can be shown that there are elements of order 16. Later it is shown that there are characters which are complex on these 2 classes and therefore both classes are of elements of order 16.

We have now shown that M(22) contains either 64, 65 or 66 conjugacy classes and the remaining fusion problems are solved while calculating the characters in the next section.

§17 Characters of M(22)

In calculating the conjugacy classes of M(22) we have incidently calculated the 2 rank 3 representations of degree 3510 and 14080.

We apply the rank 3 results of §5.

τ_{3510}: $\quad n=3510 \quad k=693 \quad l=2816 \quad \lambda=180$

$$\mu l = k(k-\lambda-1) \qquad \text{(Equation 5.3)}$$

$$\therefore \mu = 693.512/2816 = 126.$$

$$d = (180-126)^2+4(693-126) = 72^2$$

$$\{f_2,f_3\} = \frac{2k + (\lambda-\mu)(k+1) \mp \sqrt{d}(k+1)}{\mp 2\sqrt{d}} \qquad \text{(Equation 5.4)}$$

$$= \frac{2.693 + 54.3509 \mp 72.3509}{\mp 2.72}$$

$$= \{429, 3080\}.$$

$(\chi^{(22)}_{3510}, \chi^{(22)}_{14080}) = 2$ and $14080-1-3080 = 10999 \nmid |M(22)|$.

$\therefore \chi^{(22)}_{14080}$ splits into 3 characters of degrees 1,429 and 13650.

The values of these 3 characters of degrees 429,3080 and 13650 can be calculated by restricting to PSΩ(7,3) and P*.

All characters of PSU(6,2) can be regarded as characters of P* having $\{\pi\}$ in their kernel. We shall denote such characters of P* by $\chi^{(6)*}_i$ corresponding to $\chi^{(6)}_i$ of PSU(6,2).

Then $((\chi^{(6)*}_2)^{M(22)}, (\chi^{(6)*}_2)^{M(22)}) = 4.$ i.e.

this character of degree $22.3510 = 77220$ is either the sum of 4 irreducible characters of M(22) or 2 times an irreducible character. The latter is impossible as the character is odd on one conjugacy class. On restricting the character to $PS\Omega(7,3)$ it splits as :=

$4\,\chi_{2}^{(7)}$	4.78	$\chi_{8}^{(7)}$	260	$2\,\chi_{10}^{(7)}$	2.273
$3\,\chi_{15}^{(7)}$	3.1092	$2\,\chi_{22}^{(7)}$	2.2106	$2\,\chi_{28}^{(7)}$	2.4095
$2\,\chi_{34}^{(7)}$	2.5460	$\chi_{37}^{(7)}$	5824	$2\,\chi_{41}^{(7)}$	2.7280
$\chi_{47}^{(7)}$	11648	$\chi_{52}^{(7)}$	17472		

This set of 21 characters must partition into 4 subsets corresponding to the 4 irreducible characters of M(22). There are 2 conditions on this partition :=

(a) If 2 conjugacy classes of $PS\Omega(7,3)$ fuse in M(22) then the value of the character must be equal on the 2 classes.

(b) The degree of the sum of the characters in a subset of the partition must divide $2^{17}.3^{9}.5^{2}.7.11,13$.

There is only one partition consistent with these conditions and this gives 4 characters of degrees 78, 1430,32032 and 43680 of M(22). The values of the character of degree 78 can be determined on most of the conjugacy classes and indeed the values prove that one of the unresolved questions from §16 is settled, both conjugacy classes #61 and #62 exist.

The remaining characters of M(22) can be calculated by splitting the induced characters from $PS\Omega(7,3)$ and P* and characters obtained by forming tensor products of small degree.

The remaining 2 fusion problems are also resolved as assuming that the 2 pairs do not fuse one obtains characters of length $n+\frac{1}{2}$. Hence M(22) has 65 conjugacy classes and the character table of M(22) is presented as TABLE 11.

TABLE 11

CHARACTER TABLE OF M(22) OF ORDER 64,561,751,654,400

	1		2	2	2	13	13	11	11	22	22	7	14	21	5	10	10	15	20	30	3	6	6	6	12	12	12	12	24	24	3	6
	$2^{17}3^9 5^2.7.11.13$		$2^{16}.3^6.5.7.11$	$2^{17}.3^4.5$	$2^{16}.3^3$	13	13	2.11	2.11	2.11	2.11	2.3.7	2.7	3.7	$2^3.3.5^2$	$2^2.3.5$	$2^3.5$	2.3.5	$2^2.5$	2.3.5	$2^8.3^7.5.7$	$2^7.3^5.5$	$2^8.3^3$	$2^7.3^3$	$2^5.3^2$	$2^6.3^2$	$2^5.3$	$2^6.3^2$	$2^4.3$	$2^4.3$	$2^7.3^9$	$2^7.3^6$
	1		3510	1216215	36486450	4966288588800	4966288588800	2934625075200	2934625075200	2934625075200	2934625075200	1537184563200	4611553689600	3074369126400	107602919424	1076029194240	1614043791360	2152058388480	3228087582720	2152058388480	3294720	415134720	9340531200	18681062400	224172748800	112086374400	672518246400	112086374400	1345036492800	1345036492800	25625600	691891200
χ_1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	78	2.3.13	-34	14	-2	0	0	1	1	-1	-1	1	1	1	3	1	-1	0	-1	-2	15	-7	-1	1	-3	3	-1	-3	1	-1	-3	-7
χ_3	429	3.11.13	77	45	13	0	0	0	0	0	0	2	0	-1	4	2	0	1	0	-1	6	14	6	-2	2	-2	2	-2	-2	-2	24	-4
χ_4	1001	7.11.13	-231	41	-7	0	0	0	0	0	0	0	0	0	1	-1	1	1	-1	-1	56	-6	8	2	-2	4	0	-2	-2	0	29	-15
χ_5	1430	2.5.11.13	-154	86	6	0	0	0	0	0	0	2	0	-1	5	1	1	-1	-1	1	-1	-19	-1	-3	-3	-1	-1	5	-1	1	-28	8
χ_6	3003	3.7.11.13	539	59	-37	0	0	0	0	0	0	0	0	0	3	-1	-1	0	1	2	105	17	-7	-7	5	5	1	5	-1	-1	6	26
χ_7	3080	$2^3.5.7.11$	616	136	40	-1	-1	0	0	0	0	0	0	0	5	1	1	-1	-1	1	119	31	7	7	3	3	-1	3	1	1	2	22
χ_8	10725	$3.5^2.11.13$	-715	165	-43	0	0	0	0	0	0	1	-1	1	0	0	0	0	0	0	15	5	15	5	-3	-1	-1	5	3	1	114	14
χ_9	13650	$2.3.5^2.7.13$	1330	210	114	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	105	25	9	9	1	1	1	1	1	1	123	7
χ_{10}	30030	2.3.5.7.11.13	1694	526	62	0	0	0	0	0	0	0	0	0	5	-1	1	-1	1	-1	-21	29	-5	5	5	-1	-1	-7	1	-1	-102	-34
χ_{11}	32032	$2^5.7.11.13$	-2464	544	-32	0	0	0	0	0	0	0	0	0	7	1	-1	1	1	1	91	-79	-5	1	-3	-5	-1	1	-1	1	-44	20
χ_{12}	43680	$2^5.3.5.7.13$	-4256	416	-32	0	0	-1	-1	1	1	0	0	0	5	-1	1	-1	1	-1	399	-71	-1	1	-3	3	-1	-3	1	-1	-60	-44
χ_{13}	45045	$3^2.5.7.11.13$	4389	309	133	0	0	0	0	0	0	0	0	0	-5	-1	-1	1	1	-1	441	-21	9	19	-1	5	1	-1	-1	1	90	42
χ_{14}	48048	$2^4.3.7.11.13$	-1232	432	48	0	0	0	0	0	0	0	0	0	-2	-2	2	1	0	1	-84	-44	12	-12	0	-4	0	-4	0	0	258	10
χ_{15}	50050	$2.5^2.7.11.13$	-5390	450	-46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	595	-35	3	5	1	7	3	1	1	-1	73	-71
χ_{16}	50050	$2.5^2.7.11.13$	770	130	-126	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-35	5	13	-3	1	1	-3	1	-1	-1	235	-13
χ_{17}	75075	$3.5^2.7.11.13$	1155	835	131	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-210	30	-2	-10	-2	2	2	2	0	0	150	-6
χ_{18}	75075	$3.5^2.7.11.13$	7315	515	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-735	25	-1	9	1	7	-1	1	-1	1	-93	79
χ_{19}	75075	$3.5^2.7.11.13$	-5005	-125	83	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	420	-10	4	-10	-6	4	0	-2	0	2	-12	-64
χ_{20}	81081	$3^4.7.11.13$	3465	633	-87	0	0	0	0	0	0	0	0	0	6	0	-2	0	0	0	0	90	0	-6	2	0	0	-6	0	2	162	-18
χ_{21}	114400	$2^5.5^2.11.13$	-8800	480	32	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	685	-25	-3	-1	-1	1	1	-5	-1	1	28	-52
χ_{22}	138600	$2^3.3^2.5^2.7.11$	-9240	360	-24	$-\alpha$	$\alpha+1$	0	0	0	0	0	0	0	0	0	0	0	0	0	630	30	6	6	2	2	-2	2	0	0	-153	-33
χ_{23}	138600	$2^3.3^2.5^2.7.11$	-9240	360	-24	$\alpha+1$	$-\alpha$	0	0	0	0	0	0	0	0	0	0	0	0	0	630	30	6	6	2	2	-2	2	0	0	-153	-33
χ_{24}	150150	$2.3.5^2.7.11.13$	8470	70	54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	525	55	13	-9	3	5	1	-1	1	-1	57	73
χ_{25}	205920	$2^5.3^2.5.11.13$	1056	864	160	0	0	0	0	0	0	1	-1	1	-5	1	-1	1	-1	1	-279	-69	9	-5	3	1	1	-5	1	-1	-144	-24
χ_{26}	289575	$3^4.5^2.11.13$	12375	615	183	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	405	-45	21	3	-1	-3	1	3	1	-1	162	-18
χ_{27}	300300	$2^2.3.5^2.7.11.13$	-7700	1420	-84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-210	-50	-2	6	-2	2	2	2	0	0	231	49
χ_{28}	320320	$2^6.5.7.11.13$	14784	1344	192	0	0	0	0	0	0	0	0	0	-5	-1	-1	1	1	-1	406	114	6	-6	-2	-6	-2	6	0	0	-116	-12
χ_{29}	360855	$3^8.5.11$	18711	1431	279	1	1	0	0	0	0	-2	0	1	5	1	1	-1	-1	1	729	81	9	9	-3	-3	1	-3	-1	-1	0	0
χ_{30}	370656	$2^5.3^4.11.13$	-15840	1248	-96	0	0	0	0	0	0	-1	1	-1	6	0	-2	0	0	0	405	-45	21	3	-1	-3	1	3	1	-1	324	36
χ_{31}	400400	$2^4.5^2.7.11.13$	6160	1040	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-280	40	8	-8	0	0	0	0	0	0	-307	-23
χ_{32}	400400	$2^4.5^2.7.11.13$	6160	1040	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-280	40	8	-8	0	0	0	0	0	0	-307	-23
χ_{33}	400400	$2^4.5^2.7.11.13$	-18480	-240	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	980	60	-12	-4	0	4	0	4	0	0	98	-66
χ_{34}	400400	$2^4.5^2.7.11.13$	-18480	-240	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	980	60	-12	-4	0	4	0	4	0	0	98	-66
χ_{35}	450450	$2.3^2.5^2.7.11.13$	6930	470	-110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-315	45	21	-11	1	1	-3	1	-1	-1	-72	-36
χ_{36}	450450	$2.3^2.5^2.7.11.13$	25410	210	-350	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1575	-15	-9	-23	-3	3	-1	-3	1	-1	171	111
χ_{37}	576576	$2^6.3^2.7.11.13$	14784	576	-320	0	0	0	0	0	0	0	0	0	1	-1	1	[illegible]	-1	-1	126	114	-18	10	2	-6	2	6	0	0	180	-12
χ_{38}	577368	$2^3.3^8.11$	-21384	216	-72	-1	-1	0	0	0	0	1	1	1	-7	1	1	-1	1	1	729	81	9	9	-3	-3	1	-3	-1	-1	0	0
χ_{39}	579150	$2.3^4.5^2.11.13$	-6930	1230	174	0	0	0	0	0	0	-2	0	1	0	0	0	0	0	0	-405	45	-21	3	-1	3	-1	3	-1	-1	324	36
χ_{40}	582400	$2^8.5^2.7.13$	-8960	-1280	256	0	0	β	$\bar\beta$	β	$\bar\beta$	0	0	0	0	0	0	0	0	0	280	40	-8	-8	0	0	0	0	0	0	172	4
χ_{41}	582400	$2^8.5^2.7.13$	-8960	-1280	256	0	0	β	β	$\bar\beta$	β	0	0	0	0	0	0	0	0	0	280	40	-8	-8	0	0	0	0	0	0	172	4
χ_{42}	600600	$2^3.3.5^2.7.11.13$	9240	-1000	280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	210	-30	2	10	2	-2	-2	-2	0	0	-15	33
χ_{43}	600600	$2^3.3.5^2.7.11.13$	9240	-1000	280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	210	-30	2	10	2	-2	-2	-2	0	0	-15	33
χ_{44}	600600	$2^3.3.5^2.7.11.13$	21560	920	-136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	525	5	-19	5	1	-3	1	-3	1	1	-96	40
χ_{45}	675675	$3^3.5^2.7.11.13$	10395	-165	411	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	135	27
χ_{46}	720720	$2^4.3^2.5.7.11.13$	-33264	1104	-112	0	0	0	0	0	0	0	0	0	-5	1	-1	1	-1	1	1386	-54	-6	2	2	-2	-2	-2	0	0	-18	-54
χ_{47}	800800	$2^5.5^2.7.11.13$	-12320	800	-416	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-245	-35	11	13	1	3	-1	-3	-1	1	520	-8
χ_{48}	852930	$2.3^8.5.13$	-29466	1026	18	0	0	1	1	-1	-1	1	-1	1	5	-1	1	-1	1	-1	729	-81	9	-9	3	-3	1	3	-1	1	0	0
χ_{49}	938223	$3^8.11.13$	-2673	1647	207	0	0	0	0	0	0	-1	1	-1	-2	2	2	1	0	-1	-729	-81	-9	-9	3	3	-1	3	1	1	0	0
χ_{50}	972972	$2^2.3^5.7.11.13$	-24948	1836	-180	0	0	0	0	0	0	0	0	0	-3	-3	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	-243	81
χ_{51}	982800	$2^4.3^3.5^2.7.13$	15120	-240	-240	0	0	β	$\bar\beta$	$-\beta$	$-\bar\beta$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-135	-27
χ_{52}	982800	$2^4.3^3.5^2.7.13$	15120	-240	-240	0	0	$\bar\beta$	β	$-\bar\beta$	$-\beta$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-135	-27
χ_{53}	1201200	$2^4.3.5^2.7.11.13$	-30800	560	176	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	420	-20	-28	-4	0	-4	0	-4	0	0	-30	34
χ_{54}	1360800	$2^5.3^5.5^2.7$	30240	1440	288	-1	-1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	486	-54
χ_{55}	1372800	$2^7.3.5^2.11.13$	44160	640	128	0	0	0	0	0	0	2	0	-1	0	0	0	0	0	0	1560	-40	-8	8	0	0	0	0	0	0	-312	32
χ_{56}	1441792	$2^{17}.11$	0	0	0	1	1	0	0	0	0	2	0	-1	-8	0	0	-2	0	0	-512	0	0	0	0	0	0	0	0	0	640	0
χ_{57}	1791153	$3^9.7.13$	-5103	-2511	81	0	0	1	1	1	1	0	0	0	3	-3	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	1876446	$2.3^9.11.13$	37422	-1890	-306	0	0	0	0	0	0	-2	0	1	-4	2	0	-1	0	-1	729	-81	9	-9	3	-3	1	3	-1	1	0	0
χ_{59}	2027025	$3^4.5^2.7.11.13$	24255	-1455	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-90	0	6	-2	0	0	6	0	-2	-324	-36
χ_{60}	2050048	$2^{11}.7.11.13$	0	2048	0	0	0	0	0	0	0	0	0	0	-2	0	-2	-2	0	0	-1222	0	-16	0	0	0	0	0	0	0	-224	0
χ_{61}	2316600	$2^3.3^4.5^2.11.13$	-43560	-840	24	0	0	0	0	0	0	-1	1	-1	0	0	0	0	0	0	405	45	21	-3	1	-3	1	-3	1	1	-162	18
χ_{62}	2402400	$2^5.3.5^2.7.11.13$	12320	-160	160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-735	35	17	-5	-1	5	1	-1	-1	1	-384	8
χ_{63}	2555904	$2^{16}.3.13$	32768	0	0	0	0	-1	-1	-1	-1	1	1	1	4	-2	0	1	0	1	-384	-64	0	0	0	0	0	0	0	0	192	-64
χ_{64}	2555904	$2^{16}.3.13$	-32768	0	0	0	0	-1	-1	1	1	1	-1	1	4	2	0	1	0	-1	-384	64	0	0	0	0	0	0	0	0	192	64
χ_{65}	2729376	$2^5.3^9.13$	7776	-864	-288	0	0	1	1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-729	81	-9	9	-3	3	-1	-3	1	-1	0	0

	6	6	12	12	12	12	9	18	9	18	18	18	9	3	6	12	3	6	6	6	6	12	12	4	4	4	8	4	4	8	8	8	16	16
	$2^7.3^4$	$2^7.3^3$	$2^5.3^3$	$2^5.3$	$2^4.3^2$	$2^4.3^2$	2.3^4	2.3^3	$2^2.3^4$	$2^2.3^3$	$2^2.3^3$	$2^2.3^2$	3^3	$2^3.3^7$	$2^3.3^3$	$2^2.3^2$	$2^6.3^7$	$2^4.3^5$	$2^5.3^3$	$2^6.3^3$	$2^4.3^3$	$2^3.3^2$	$2^4.3^2$	$2^{10}.3^2.5$	$2^{10}.3$	$2^9.3^2$	2^5	$2^{12}.3^3$	$2^{12}.3$	$2^7.3$	$2^7.3$	2^7	2^5	2^5
	6227020800	18681062400	74724249600	672518246400	448345497600	448345497600	398529331200	1195587993600	199264665600	597793996800	597793996800	1793381990400	2391175987200	3690086400	298896998400	1793381990400	461260800	16605388800	74724249600	37362124800	149448499200	896690995200	448345497600	1401079680	21016195200	14010796800	2017554739200	583783200	5254048800	168129561600	168129561600	504388684800	2017554739200	2017554739200
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	5	1	-3	1	-1	-1	0	2	3	-1	-1	-1	0	-3	1	-1	6	2	2	-2	-2	0	2	-6	2	2	0	6	-2	2	-2	2	0	0
χ_3	0	4	4	0	2	2	3	-1	0	2	2	0	0	-3	1	-1	15	5	3	7	1	-1	-1	5	5	5	1	13	-3	1	1	1	-1	-1
χ_4	5	-7	1	-3	-1	-1	2	0	2	0	0	2	-1	2	2	2	2	-6	2	2	2	-2	2	-11	-3	5	-1	1	9	-3	1	1	1	1
χ_5	-4	0	-4	0	-2	-2	-1	-1	-1	-1	-1	-1	2	-1	3	1	26	8	2	-6	0	0	-2	-6	2	10	0	14	6	-2	+2	2	0	0
χ_6	14	2	2	-2	0	0	0	2	3	-1	-1	-1	0	6	2	0	15	-1	-1	-1	-1	-1	3	11	-5	3	-1	11	-5	-1	-1	3	-1	-1
χ_7	10	-2	6	2	0	0	-1	1	5	1	1	1	-1	2	-2	0	20	4	4	4	4	0	0	24	8	0	0	24	8	4	4	0	0	0
χ_8	-6	-10	2	-2	0	0	3	-1	0	2	2	0	0	6	2	0	24	-4	0	8	-4	0	0	-15	-7	9	-1	29	-11	1	-3	1	-1	-1
χ_9	3	15	-5	-1	1	1	3	1	0	-2	-2	0	0	15	3	1	33	7	-3	9	3	1	1	10	10	10	2	-14	2	-2	-2	-2	0	0
χ_{10}	-14	-10	2	-2	0	0	-3	-1	-3	-1	-1	1	0	6	2	0	60	2	4	-4	2	2	0	26	2	18	0	38	-2	+2	-2	2	0	0
χ_{11}	4	4	-8	0	0	0	1	-1	1	-1	-1	1	-2	10	-2	0	64	2	4	-8	-2	0	0	-24	8	0	0	64	0	4	-4	0	0	0
χ_{12}	20	4	0	0	0	0	3	1	3	1	1	-1	0	-6	-2	0	48	10	-4	-8	-2	0	0	-24	8	0	0	0	0	-4	4	0	0	0
χ_{13}	-6	10	2	2	-2	-2	0	0	0	0	0	0	0	9	1	1	-18	6	6	-2	-2	2	-2	41	1	1	-1	29	5	1	5	1	-1	-1
χ_{14}	18	-6	2	2	-2	-2	3	1	0	-2	-2	0	0	-12	0	-2	6	-8	6	6	0	0	-2	0	0	16	0	-16	-16	0	0	0	0	0
χ_{15}	9	-7	1	1	1	1	-2	-2	4	-2	-2	0	1	19	-1	1	10	-8	6	2	-4	-2	-2	-50	6	-2	0	10	-14	2	-2	-2	0	0
χ_{16}	-5	3	-5	3	3	3	1	-1	1	-1	-1	1	1	-8	0	0	19	5	-5	3	-3	1	3	10	-6	18	-2	-14	18	2	2	-2	0	0
χ_{17}	-2	2	2	-2	0	0	0	0	-6	0	0	-2	0	-12	-4	0	51	3	7	11	-1	1	3	-5	11	3	-1	83	19	3	3	-1	1	1
χ_{18}	11	-9	-5	-1	1	1	0	-2	3	1	1	-1	0	-12	0	-2	-3	-11	5	-3	-3	1	1	55	-1	7	1	-5	-13	-5	-1	-1	1	1
χ_{19}	28	8	4	0	2	2	-3	-1	-3	-1	-1	1	0	15	-1	-1	42	-10	-2	2	2	0	2	15	-9	-1	1	-5	3	-1	3	3	-1	-1
χ_{20}	-6	6	6	2	0	0	0	0	0	0	0	0	0	0	0	0	81	9	-3	9	-3	-1	-3	5	-3	-3	-1	33	-7	-7	-3	1	1	1
χ_{21}	12	-4	-8	0	0	0	-2	2	4	2	2	0	1	-26	2	0	-8	2	0	8	2	2	0	-40	-8	0	0	64	0	4	-4	0	0	0
χ_{22}	-9	15	-1	-1	-1	-1	0	0	0	0	0	0	0	9	-3	-1	-45	3	3	3	3	-1	-1	-40	-8	8	0	8	8	0	0	0	0	0
χ_{23}	-9	15	-1	-1	-1	-1	0	0	0	0	0	0	0	9	-3	-1	-45	3	3	3	3	-1	-1	-40	-8	8	0	8	8	0	0	0	0	0
χ_{24}	25	9	-7	1	-3	-3	0	-2	-3	1	1	1	0	-24	0	0	48	-8	-8	0	0	0	0	-30	10	-6	0	-34	22	2	-2	2	0	0
χ_{25}	0	-8	-8	0	0	0	0	0	0	0	0	0	0	18	-2	0	72	12	0	-8	4	0	0	24	-8	0	0	64	0	-4	4	0	0	0
χ_{26}	-6	6	6	2	0	0	0	0	0	0	0	0	0	0	0	0	-81	9	3	-9	-3	-1	3	35	-5	3	1	15	23	-1	-5	3	1	1
χ_{27}	-11	9	5	1	-1	-1	-3	1	-3	1	1	1	0	-21	3	-1	114	4	-2	-6	0	-2	2	-20	-4	20	0	-4	-20	0	0	-4	0	0
χ_{28}	12	-12	0	0	0	0	-2	0	4	0	0	0	1	-8	0	0	64	6	0	0	6	-2	0	16	16	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39	7	-9	-1	-9	-9	-1	-1	-1	-1	-1
χ_{30}	-12	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-18	0	0	6	2	0	-40	-8	0	0	0	0	-4	4	0	0	0
χ_{31}	5	1	-3	1	7	-5	-1	1	-1	1	+1	-1	-1	17	1	1	-10	-14	2	-2	-2	0	-2	0	0	16	0	-48	16	0	0	0	0	0
χ_{32}	5	1	-3	1	-5	7	-1	1	-1	1	+1	-1	-1	17	1	1	-10	-14	2	-2	-2	0	-2	0	0	16	0	-48	16	0	0	0	0	0
χ_{33}	-6	-10	-2	2	0	0	2	0	-4	0	0	0	-1	-10	2	0	-28	6	0	-4	2	0	0	0	0	0	0	16	-16	0	0	0	$2\sqrt{2}i$	$-2\sqrt{2}i$
χ_{34}	-6	-10	-2	2	0	0	2	0	-4	0	0	0	-1	-10	2	0	-28	6	0	-4	2	0	0	0	0	0	0	16	-16	0	0	0	$-2\sqrt{2}i$	$2\sqrt{2}i$
χ_{35}	0	4	4	0	-2	-2	0	0	0	0	0	0	0	9	1	1	9	-9	-3	1	-5	1	1	10	-6	-14	2	130	-30	2	2	-2	0	0
χ_{36}	3	7	3	-1	1	1	0	0	0	0	0	0	0	9	1	1	-18	12	6	-2	4	0	-2	30	-10	-2	0	-6	2	2	-2	-2	0	0
χ_{37}	36	4	0	0	0	0	0	0	0	0	0	0	0	18	-2	0	72	6	0	-8	-2	2	0	-16	-16	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	-8	0	0	72	24	-4	-4	0	0	0
χ_{39}	-12	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81	9	-3	9	-3	-1	-3	-10	-10	6	2	-18	30	2	2	2	0	0
χ_{40}	-20	4	0	0	0	0	1	1	1	1	1	1	1	10	-2	0	64	-14	4	-8	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	-20	4	0	0	0	0	1	1	1	1	1	1	1	10	-2	0	64	-14	4	-8	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	17	1	1	1	1	1	0	0	-3	$3+6\omega$	$-3-6\omega$	-1	0	-15	1	1	57	-3	5	1	1	-1	1	-40	-8	-8	0	-8	-8	0	0	0	0	0
χ_{43}	17	1	1	1	1	1	0	0	-3	$-3-6\omega$	$3+6\omega$	-1	0	-15	1	1	57	-3	5	1	1	-1	1	-40	-8	-8	0	-8	-8	0	0	0	0	0
χ_{44}	-16	8	0	0	0	0	3	-1	3	-1	-1	-1	0	-42	2	0	12	-22	-4	-4	2	-2	0	-40	-8	0	0	72	24	4	4	0	0	0
χ_{45}	15	3	3	-1	1	1	0	0	0	0	0	0	0	54	6	-2	-54	0	-6	-6	0	0	-1	-45	3	-5	-1	75	11	3	3	-1	1	1
χ_{46}	6	2	10	-2	0	0	0	0	0	0	0	0	0	-18	2	0	36	0	0	-4	-4	2	0	-16	16	0	0	80	16	0	0	0	0	0
χ_{47}	8	-8	0	0	0	0	1	1	-5	1	1	-1	1	34	-2	0	16	-8	-4	-8	4	-2	0	40	8	0	0	0	0	4	-4	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-2	-18	0	-54	18	-2	2	-2	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	-1	-9	-1	63	15	-5	-5	-1	-1	-1
χ_{50}	-27	9	-3	1	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-36	12	-12	0	60	-20	0	0	4	0	0
χ_{51}	-15	-3	-3	1	-1	-1	0	0	0	0	0	0	0	27	3	-1	54	0	6	6	0	0	2	0	0	-16	0	-48	16	0	0	0	0	0
χ_{52}	-15	-3	-3	1	-1	-1	0	0	0	0	0	0	0	27	3	-1	54	0	6	6	0	0	2	0	0	-16	0	-48	16	0	0	0	0	0
χ_{53}	2	2	2	2	-2	-2	0	-2	3	1	1	-1	0	51	-1	1	-48	-2	-4	8	2	0	4	0	0	16	0	-16	-16	0	0	0	0	0
χ_{54}	-18	18	-6	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-96	-32	0	0	0	0	0
χ_{55}	-8	-16	0	0	0	0	0	2	-3	-1	-1	1	0	12	-4	0	-24	-4	-8	8	-4	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	-2	0	4	0	0	0	-2	-8	0	0	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	9	9	1	81	33	-3	-3	-3	-1	-1
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-30	10	18	0	54	-18	-2	2	2	0	0
χ_{59}	+12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81	-9	-3	9	3	1	-3	85	-3	-3	-1	9	-15	1	-3	1	1	1
χ_{60}	32	0	0	0	0	0	4	0	-2	0	0	2	1	-8	0	0	-80	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	6	-6	-6	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	18	0	0	-6	-2	0	40	-8	0	0	-24	-8	4	4	0	0	0
χ_{62}	-16	-8	8	0	0	0	3	-1	3	-1	-1	-1	0	-6	-2	0	48	8	-4	-8	4	2	0	-40	-8	0	0	-64	0	4	-4	0	0	0
χ_{63}	0	0	0	0	0	0	-3	-1	0	2	2	0	0	-24	0	0	-96	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	-3	1	0	-2	-2	0	0	-24	0	0	-96	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	8	0	0	0	0	-4	4	0	0	0

CHAPTER 3 A CHARACTERIZATION OF M(22)

§18 Statement of the problem

In this chapter we characterize M(22) by the structure of the centralizer of an involution method. Throughout the next three sections G will denote an arbitrary simple group containing an involution π such that that $C_G(\pi) \cong$ centralizer of a 3-transposition in M(22).

Now $C_G(\pi)/\langle\pi\rangle \cong PSU(6,2)$ and this extension of $C_2 = \langle\pi\rangle$ by PSU(6,2) is non-splitting.

The involutions in $C_G(\pi) = P^*$ (see § 16) are as follows :=

name	$\lvert C_{P^*}(X)\rvert$	# of conjugates in P*	SQ or NSQ
π	$2^{16}.3^6.5.7.11$	1	NSQ
d	$2^{16}.3^4.5$	$3^2.7.11 = 693$	NSQ
πd	$2^{16}.3^4.5$	$3^2.7.11 = 693$	SQ
t	$2^{15}.3^2$	$2.3^4.5.7.11 = 62370$	SQ
πt	$2^{15}.3^2$	$2.3^4.5.7.11 = 62370$	SQ
n	$2^{12}.3^2$	$2^4.3^4.5.7.11 = 498960$	SQ

The 1st column gives the name where d, $t = d_1d_2$ and $n = d_1d_2d_3$ are representatives of the conjugacy classes of involutions in PSU(6,2).

If SQ is written in the 4th column then there is y in P* such that $y^2 = x$. NSQ means that there is no such y of order 4.

<u>Lemma 18.1</u> π and d are NSQ.

πd, t, πt and n are SQ.

<u>Proof</u> In $M(22)$ $\pi \sim d$. Consider a $d \in L_{10} \leq N_{22}$. Then a Sylow 2-subgroup of $C_{M(22)}(d)$ is contained in N_{22}. No element in this Sylow 2-subgroup has square equal to d. Therefore π and d are NSQ.

Let π x y a b c be a hexad in $S(3,6,22)$. $M(22)$ contains involutions of the type $1^6 2^8$ and the action of the stabilizer of a hexad in $M(22)$ on the hexad is isomorphic to A_6. (Todd [10].) Therefore there is a permutation $(\pi)(x\ a)(y\ b)(c) \in M_{21} \leq P^*$. Call this element i. Then $\pi+x+y.i \in P^*$ and $(\pi+x+y.i)^2 = \pi+c$. Therefore there are elements of the type πd are SQ.

Similarly one can prove that t, πt and n are SQ. Also by working in the subgroup of $N_{22} \leq P^*$ it is easy to see that $n \sim \pi n$ in P^*. ЧЧ

§19 Fusion of involutions

Lemma 19.1 In G $\pi \sim d$, and no other involution in P^* fuses to π in G. We call this conjugacy class D.

Proof Clearly t, πt, πd and n cannot fuse to π as they are all SQ even in P^* and π is a NSQ in G.

Glauberman's Lemma [5, Corollary 1]

Let S be a Sylow 2-subgroup of a finite group G. Suppose $x \in S$. A necessary and sufficient condition for $x \notin Z^*(G)$ (= the inverse image in G of $Z(G/O(G))$.) is that there is a $y \in C_S(x)$ such that y is conjugate to x in G and $y \neq x$. ◁◁

That is, if G is simple then every involution fuses to another in its centralizer. $\therefore \pi \sim d$ in G. ◁◁

Lemma 19.2 $|Syl_2(G)| = 2^{17}$.

Proof $(1_{P^*})^G(\pi) = 1 + 693 = 694 \equiv 2 \pmod 4$.

$\therefore (1_{P^*})^G(1) \equiv 2 \pmod 4$ as G is simple and in the permutation representation of G on the cosets of P^* every element must be an even permutation, in particular π is an even permutation.

$\therefore |G : P^*| \equiv 2 \pmod 4$

$\therefore |Syl_2(G)| = 2^{17}$. ◁◁

Now $|Syl_2(C_G(\pi d))| \geq 2^{16}$ and as πd is an even permutation in the above representation $(1_{P^*})^G(\pi d) \equiv 2\ (4)$.

$\therefore |Syl_2(C_G(\pi d))| = 2^{17}$. i.e. πd is a central involution in G, that is an involution central in a Sylow 2-subgroup. Let $T = Syl_2(C_{P*}(\pi d))$. Let S be a Sylow 2-subgroup of $C_G(\pi d)$ containing T. Then $T \triangleleft S$.

<u>Lemma 19.3</u> L_{10} ch T and therefore $L_{10} \triangleleft S$.

<u>Proof</u> $M_{24} \geqslant (C_2)^4.A_8$ a splitting extension of an elementary abelian group of order 16 by A_8 with A_8 acting as $PSL(4,2)$ on the v.s. $M_{21} \geqslant (C_2)^4.A_5$ with the A_5 acting transitively on the 15 involutions. (The transitivity is clear from the permutation character of A_8 of degree 15, see Littlewood [13] p. 276.)

Let α be any involution in M_{21}. α is of permutation type $1^5 2^8$. Therefore using our knowledge of the elements in L_{10}, (see §3) we have :=

$$|C_{L_{10}}(\alpha)| = 1 + 6 + \binom{6}{2} + 8 + \tfrac{1}{2}.\binom{6}{3} + 12.2 = 2^6.$$

(12 = 13 - 1. An involution α fixes exactly 13 blocks in S(3,6,22). See character table of M_{22}, Burgoyne and Fong [2].)

Consider $(C_2)^4.A_5 = L_4.A_5$. If $y \in Syl_2(A_5)$ then $|C_{L_4}(y)| = 4$. (Character table of A_8, Littlewood [13].) $\therefore$ a maximal elementary abelian subgroup of a $Syl_2(M_{21})$ has order 2^4.

However it is clear that $|C_{L_{10}}(\alpha) \cap C_{L_{10}}(\beta)| \leqslant 1 + 6 + \binom{6}{2} + \tfrac{1}{2}.\binom{6}{3} = 2^5$ for all α, β in L_4. $\therefore$ There is no elementary abelian subgroup of T of order 2^{10} containing α. i.e. L_{10} is the only elementary abelian subgroup of T of order 2^{10}. $\therefore L_{10}$ ch T. 44

<u>Lemma 19.4</u> $N_G(L_{10})/L_{10} \cong M_{22}$, i.e. $N_G(L_{10}) \cong N_{22}$

<u>Proof</u> Let $x \in S\backslash T$. x does not fix π in its action by conjugation on L_{10}.

Therefore $N_G(L_{10})/L_{10}$ contains a transitive extension of M_{21}. This can only be M_{22} (see Luneberg [8]). ##

It is now clear that $t \sim \pi d$ and $n \sim \pi t$ in G. Suppose $\pi d = d_1 d_2 d_3$ as a product of a commuting triple of elements in D. Let S be a Sylow 2-subgroup of $C_G(\pi d)$ containing d_1, d_2 and d_3. Now $|S:S \cap C_G(\pi)| = 2$, therefore at least one of d_1, d_2 or $d_3 \in C_G(\pi)$; say d_1.

Then πd, π, d, d_1, d_2, d_3 all lie in $C_G(d_1) \cong P^*$. This is a contradiction as in P^* no "t" is an "n".

Hence G has exactly 3 conjugacy classes of involutions which we call D, T and N. (Every involution in $C_G(\pi d)\backslash C_G(\pi)$ fuses to one in $C_G(\pi)$ under conjugation by M_{22}.)

Since n is an even permutation on 3510 symbols, $|Syl_2(C_G(n))| = 2^{16}$. Hence we have :=

$d \in D \quad |C_G(d)| = 2^{16}.3^6.5.7.11$

$t \in T \quad |C_G(t)| = 2^{17}.3^4.5.m_1$

$n \in N \quad |C_G(n)| = 2^{16}.3^2.m_2$ where m_1 and m_2 are odd integers.

<u>Lemma 19.5</u> $m_1 = 1$.

<u>Proof</u> Suppose $\pi d = d_1 d_2$. In P^* we know that if $\pi d = d_1 d_2$ then $\{d_1, d_2\} = \{\pi, d\}$. Therefore $d_1, d_2 \in C_G(\pi d) \setminus P^*$. $d_1 \in$ a Sylow 2-subgroup of $C_G(\pi d)$ which without loss of

generality $\leqslant N_{22}$. $d_1 = p.q$ where $p \in L_{10}$ and q is of the form $1^6 2^8$ in M_{22}. Therefore there is a $d_3 \in L_{10}$, $d_3 \in C_G(d_1)$. Thus πd, π, d, d_1, d_2 all $\in C_G(d_3)$. Therefore $\{\pi, d\} = \{d_1, d_2\}$.

Hence we have that every element in the conjugacy class T is uniquely expressible as $t = d_1 d_2 = d_2 d_1$. $C_G(\pi)$ contains $693 + 1$ elements in D.

$$\therefore \left(\frac{|G|}{2^{16}.3^6.5.7.11} \times 693 \right) \times \tfrac{1}{2} = \frac{|G|}{2^{17}.3^4.5.m_1}$$

$\therefore \quad m_1 = 1.$ ◁◁

<u>Lemma 19.6</u> $m_2 = 3$.

<u>Proof</u> In N_{22} and also P^* we have $n = d_1 d_2 d_3 = d_4 d_5 d_6$ implies that $\{d_4, d_5, d_6\} = \{d_1, d_2, d_3\}$ or $\{d_4, d_5, d_6\}$ is another uniquely determined triple of elements in D.

Now suppose $n = \pi d_2 d_3 = d_4 d_5 d_6$. $|\mathrm{Syl}_2(C_G(n)) : \mathrm{Syl}_2(C_G(n) \cap C_G(\pi))| = 2$. Therefore one of d_4, d_5, d_6 say $d_4 \in C_G(\pi)$. Then π, d_2, d_3, d_4, d_5, d_6 all $\in C_G(d_4)$. ($d_2 d_3 \in C_G(d_4)$ implies that $d_2, d_3 \in C_G(d_4)$.)

Hence every $n \in N$ can be expressed as the product of exactly 2 different commuting triples of elements in D.

$$\therefore \left(\frac{|G|}{2^{16}.3^6.5.7.11} \times 693 \times 180 \right) \frac{1}{3!} \times \tfrac{1}{2} = \frac{|G|}{2^{16}.3^2.m_2}$$

$\therefore \quad m_2 = 3.$

§20 Group Order

Lemma 20.1 (Thompson: see Held [6], p.279.)

Let G be a group with exactly 3 conjugacy classes of involutions D, T and N with representatives d, t and n.

Let I be the set of all involutions in G.

For any involution $\rho \in G$ let $a(\rho) = \{\#(\alpha,\beta) \mid \alpha \in D, \beta \in T, \rho = (\alpha\beta)^i$ for some integer $i\}$. Then :=

$$|G| = |C_G(t)|.a(d) + |C_G(d)|.a(t) + \frac{|C_G(d)||C_G(t)|}{|C_G(n)|}.a(n).$$

Proof Let $J = \{(\alpha,\beta) \mid \alpha \in D, \beta \in T\}$. Now $(\alpha\beta)^n = 1$ for odd n implies that $\alpha \sim_G \beta$. Hence for all $(\alpha,\beta) \in J$ $|\alpha\beta|$ is even, i.e. every $(\alpha,\beta) \in J$ corresponds to a unique involution in I.

$$\therefore |J| = \frac{|G| . |G|}{|C_G(d)||C_G(t)|} = \sum_{\rho\in I} a(\rho) =$$

$$= \frac{|G|}{|C_G(d)|}.a(d) + \frac{|G|}{|C_G(t)|}.a(t) + \frac{|G|}{|C_G(n)|}.a(n). \quad \#\#$$

Hence $|G| = 2^{17}.3^4.5.a(d) + 2^{16}.3^6.5.7.11.a(t) + 2^{17}.3^7.5^2.7.11.a(n)$.

Lemma 20.2 $a(n) = 6$.

Proof Clear from proof of Lemma 19.6. For if $n = d't'$ implies $n = d'(d_1'd_2')$. $\{d',d_1',d_2'\} = \{d_1,d_2,d_3\}$ or $\{d_4,d_5,d_6\}$.

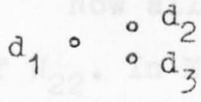
<u>Lemma 20.3</u> If $|\alpha\beta| = 2n$ and $(\alpha\beta)^n = \rho$ then $\alpha, \beta \in C_G(\rho)$.

<u>Proof</u> Clear. 44

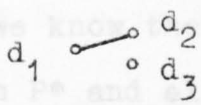
<u>Lemma 20.4</u> $a(\pi)$ $(= a(d)) = 513 \times 693$.

<u>Proof</u> By lemma 20.3 we need only consider involutions in $C_G(\pi) = P^*$. The d's in P^* behave as 3-transpositions.

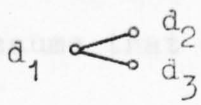
Now if $t' = d_2 d_3$ and $d' = d_1$ we have :=



The involution is a d if and only if $d_1 = d_2$ or $d_1 = d_3$. Otherwise the involution is an n.



The involution is a $d = d_3$.



The involution is a t.

We use the table of involutions in P^* given at the beginning of §18 and see that there are 693 solutions to the first diagram and 512×693 solutions to the second diagram, i.e. $d_3 = \pi$, d_2 takes 693 values and d_1 takes 512 values for each value of d_2. 44

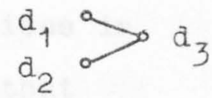
<u>Lemma 20.5</u> $a(\pi d)$ $(= a(t)) = 2304$.

<u>Proof</u> We take πd as our representative of the conjugacy class T. We seek pairs d', t' in $C_G(\pi d)$ such that $(d't')^n = \pi d$ for some integer n (Lemma 20.3).

Let $M = C_G(\pi d) \cap C_G(\pi)$.

First suppose that $d', t' \in M$. Then we are in a conjugate of P^* and we may use the diagrams in Lemma 20.4.

Hence we may suppose that $t' = d_1 d_2$ and $d' = d_3$



and $d_1 d_3 \neq d_3 d_1$ and $d_2 d_3 \neq d_3 d_2$.

Then $(t'd')^2 = d_1 d_2 d_3 d_1 d_2 d_3 = (d_3)^{d_1 d_2}\, d_3$.

But then if $d_3 (d_3)^{d_1 d_2} = \pi d$ we have by Lemma 19.5 that $d_3 = d$ or $d_3 = \pi$. But then $t' \notin M$ contrary to assumption.

Hence either d' or t' does not belong to M.

Now all involutions in $C_G(\pi d) \setminus M$ lie in a conjugate of N_{22}. In $N_{22} \setminus L_{10}$ all involutions belong to T or N (we know the involutions in one special M_{21} as it lies in P^* and all M_{21}'s in N_{22} are conjugate). Hence we may assume that $d' \in M$ and $t' \notin M$.

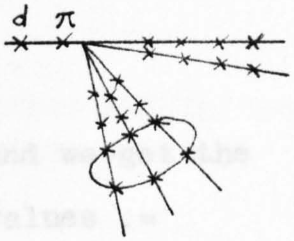
Let $(d')^{t'} = d'' \in M$. Therefore $|d'd''| = 1, 2$ or 3 as $M \leqslant P^*$. If $|d'd''| = 1$ then $d' = d''$ and involution associated with d', t' is $d't'$. $d't'$ does not belong to M and thus cannot be πd. If $|d'd''| = 3$ then involution associated with d', t' is $(d't')^3 = d't'd't'd't' = (t')^{d'}.d'' \notin M$ and thus cannot be πd. Hence $|d'd''| = 2$ and $d'd''$ is the involution associated with d', t'. Thus $d' = d$ or π and t' does not commute with d. Conversely if $t' \in C_G(\pi d) \setminus M$ then $\pi^{t'} = d$ and $(\pi t')^2 = \pi t' \pi t' = \pi d$.

Therefore $a(\pi d) = 2\psi$ where ψ = the number of t's in $C_G(\pi d) \setminus C_G(\pi)$.

Let $i \in 1^6 2^8$ the conjugacy class of involutions in $M_{22} \leqslant N_{22}$. From the proof of Lemma 19.3 we have that $|C_{L_{10}}(i)| = 2^6$. Hence there are 64.1155 involutions in $N_{22} \backslash L_{10}$. (There are $1155 = 3.5.7.11$ involutions in M_{22} with centralizer of order $2^7.3$. See Burgoyne and Fong [2].) $|C_{N_{22}}(i)| = 2^6.2^7.3$ and $|C_{N_{22}}(id'')| = 2^6.2^7$ where d'' is an involution in L_{10} fixed by i. This accounts for $2^4.1155 + 2^4.3.1155$ involutions, i.e. all involutions in $N_{22} \backslash L_{10}$. Now $i \in T$, $id'' \in N$ (as above by conjugation into P^* by elements of M_{22}).

We now find all conjugacy classes of involutions in $N_{22} \cap C_G(\pi d)$ which fuse to T in G and which do not centralize either π or d. There are precisely $2^4.1155 \times \frac{8}{11.21} = 5.2^7$ elements satisfying this condition. (An involution in M_{22} is of shape $1^6 2^8$ and therefore interchanges 8 pairs of points and there are altogether $\binom{22}{2} = 11.21$ pairs of points.)

Let i be one of these elements. Let $K = N_{22} \cap C_G(\pi d)$ $= L_{10}.\overline{M}_{20}$, where $\overline{M}_{20}$ is the subgroup of M_{22} containing M_{20}, $|\overline{M}_{20} : M_{20}| = 2$.



i fixes a block which meets 3 of the blocks through d and π in 2 points. Any element in $C_{\overline{M}_{20}}(i)$ must fix this block and therefore $|C_{\overline{M}_{20}}(i)|$ $\leqslant 2.|C_{A_6}(i)| \leqslant 2.\frac{2.2.2.3!}{2} = 2^4.3$.

Therefore $|C_K(i)| \leqslant 2^{10}.3$. Thus all elements of the same form as i are conjugate and $|C_K(i)| = 2^{10}.3$.

Now $|C_G(\pi d) : K| = 27$ and hence $|C_{C_G(\pi d)}(i)| =$ $|C_K(i)| \times l$ where l is odd and $\leqslant 27$.

i.e. l=1,3,5,9 or 15.

If l = 15 then $|C_{C_G(\pi d)}(i)| = 2^{10}.3^2.5$

$\#$ of conjugates of $i = \psi = 2^7.3^2 = 1152$.

Therefore we assume that $l \neq 15$, i.e. l=1,3,5 or 9.

Using Thompson's group order formula we have :=

l=1	$\lvert G\rvert = 2^{17}.3^8.5.7.11.1987$	$\lvert D\rvert = 35766$
l=3	$\lvert G\rvert = 2^{17}.3^8.5.7^2.11.101$	$\lvert D\rvert = 12726$
l=5	$\lvert G\rvert = 2^{17}.3^8.5.7.11^2.41$	$\lvert D\rvert = 8118$
l=9	$\lvert G\rvert = 2^{17}.3^7.5.7.11.29^2$	$\lvert D\rvert = 5046$

Now when l=1 we have by Sylow's theorem that n_{1987} the number of Sylow 1987-subgroups must be 1 and thus G will not be simple.

l=3, l=5 and l=9 may be eliminated by character arguments. As an example suppose that l=9, $|D| = 5046$. Then G has a permutation representation of degree 5046 with values :=

1	D	T	N
5046	694	182	54

We restrict this character to P* and we get the associated character of PSU(6,2) with values :=

1	D	T	N
2870	694	182	54

$2870 \equiv 10 \pmod{11}$. As no character of PSU(6,2) of degree < 2870 has value +1 on elements of order 11 the representation on $|D|$ must be of rank $\geqslant 10$ as the principal character must occur with multiplicity $\geqslant 10$. This is easily seen to be impossible from the character table of PSU(6,2), TABLE 5. 44

Hence we have that $|G| = 2^{17}.3^{4}.5.513.693 + 2^{16}.3^{6}.5.7.11.2304 + 2^{17}.3^{7}.5^{2}.7.11.6 = 64,561,751,654,400$ $= 2^{17}.3^{9}.5^{2}.7.11.13$.

Thus $|D| = 3510 = 2.3^{3}.5.13$. As described above this yields a character of PSU(6,2) with values 2102, 694, 182 and 54 on the conjugacy classes 1, D, T and N. Inspection of the character table of PSU(6,2) shows that the only character with these values contains the principal character with multiplicity 3. Therefore G acts as a rank 3 permutation group on the conjugacy class D. Hence D is a conjugacy class of 3-transpositions, and by Theorem 1.2 we have $G \cong M(22)$.

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